# Nonlinear Diamagnetic Stabilization Effects on $m = 2$, $n = 1$ Cylindrical Double-Tearing Modes in Hall MHD Simulations

Stephen Abbott and Kai Germaschewski
Department of Physics, University of New Hampshire, Durham, NH

University of New Hampshire

## Introduction

Reversed-shear tokamak configurations have some promising features for improved stability and confinement but may be unstable to Double-Tearing Modes (DTMs). Nonlinearly these instabilities can potentially cause significant plasma motion and disruptions of the annular current ring [2, 3]. Recent research has shown that equilibrium shear flows can have a stabilizing effect on both linear and nonlinear DTMs. If the rotation between the two tearing surfaces is large enough compared to the growth rate ($\Delta\omega \gtrapprox \gamma$) they cannot couple linearly and the system collapses to two localized eigenmodes [6], though they may re-couple nonlinearly.

Diamagnetic drifts (characteristed by the frequency $\omega_*$) have the potential to provide both differential rotation and additional reconnection-layer stabilization [7]. Internal Transport Barriers (ITBs) with significant pressure gradients are frequently observed in reversed-shear configurations [5], suggesting $\omega_*$ effects are a likely candidate for stabilization. We extend our previous work on linear $\omega_*$ stabilization into the nonlinear regime of an $m = 2$, $n = 1$ DTM in cylindrical geometry using the extended MHD code `MRC-3D`. While we do find evidence of nonlinear stabilization, we find the effectiveness is highly dependent on the location of the pressure profile.

## Equilibrium

We use the non-monotic safety factor profile from Ref [4]:

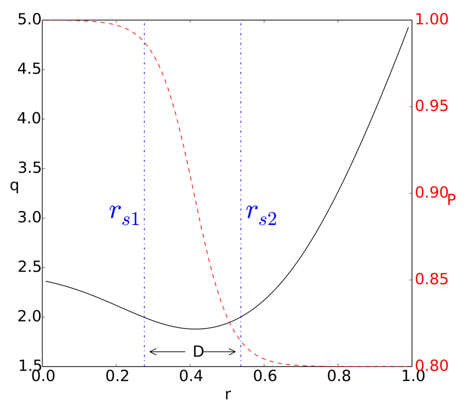
$$q(r) = q_0 F_1(r)\left\{1 + (r/r_0)^{2w(r)}\right\}^{1/w(r)} \qquad r_0 = r_A\left|[m/(nq_0)]^{w(r_A)} - 1\right|^{-1/[2w(r_A)]}$$

$$w(r) = w_0 + w_1 r^2 \qquad F_1(r) = 1 + f_1 \exp\left\{-[(r - r_{11})/r_{12}]^2\right\}$$

with the constant values: $r_A = 0.655$, $w_0 = 3.8824$, $w_1 = 0$, $f_1 = -0.238$, $r_{11} = 0.4286$, $r_{12} = 0.304$, $m = 2$, $n = 1$. $q_0$ may be varied near 2.5 to change the separation $D$ between two $q = 2$ surfaces. Assuming $B_{z0} = R_{major} = 10$ we find the in-plane field $B_\theta$. For this work we fix $q_0 = 2.5$, giving $D \approx 0.26$. Density profiles are of the form [5]:

$$N(r) = N_0\left\{1 - (1 - N_b)\frac{\tanh(r_0/\delta_N) + \tanh[(r - r_0)/\delta_N]}{\tanh(r_0/\delta_N) + \tanh[(1 - r_0)\delta_N]}\right\}$$

Where $N_0 = 1$, $r_{s1} \le r_0 \le r_{s2}$, and $r_{s1(2)}$ is the inner (outer) $q = 2$ surface. The parameters $\delta_N$, $N_b$, and $r_0$ are chosen based on the desired diamagnetic drifts ($\omega_*$) at each tearing surface. Temperature $T = 1$ is uniform.




## MRC-3D Model

$$\mathbf{E} = -\mathbf{v}_i \times \mathbf{B} + \frac{d_i}{\rho}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + \eta\mathbf{J} - \eta_2\nabla^2\mathbf{J}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v} - D\nabla\rho) = 0$$

$$\frac{\partial\rho\mathbf{v}}{\partial t} + \nabla\cdot[\rho\mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + \mathbf{I}(p + B^2/2) - \rho\nu\nabla\mathbf{v}] = 0$$

$$\frac{\partial T_e}{\partial t} + \mathbf{v}\cdot\nabla T_e + (\gamma - 1)T_e\nabla\cdot\mathbf{v} = 0$$

$$p_s = \rho T_s, \quad p = p_e + p_i = (1 + \tau)\rho T_e$$

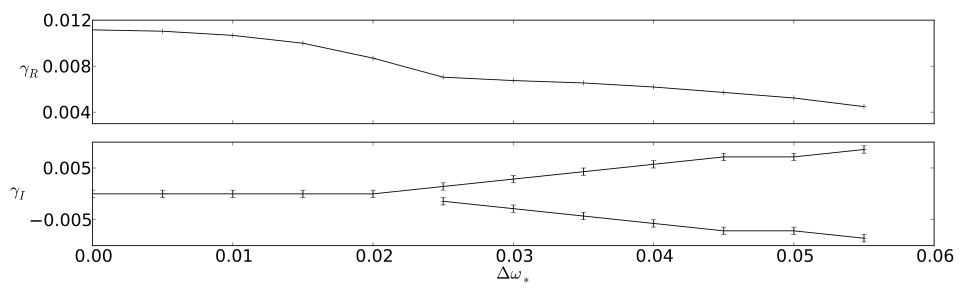
Where $\tau = T_i/T_e$, $\gamma = 5/3$ for an adiabatic equation of state, $\eta$ is the resistivity, $d_i$ is the ion inertial length, $\nu$ is the fluid viscosity, and $D$ is a particle diffusivity parameter. For this work we fix $d_i = 0.1$, $\eta = 2e - 5$, and $\tau = 0$. The other dissipation parameters are given small values to aid numerical stability. Faraday's Law is used to evolve $\mathbf{B}$.

With these parameters, the diamagnetic drift frequency is given by:

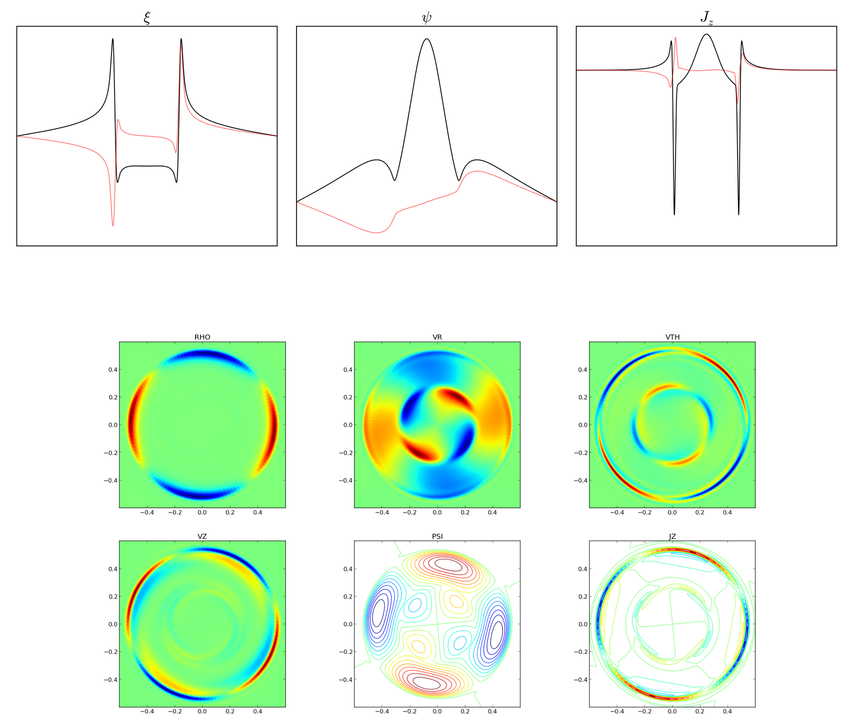
$$\omega_* = \left[d_i\frac{\nabla p_e \times \mathbf{B}}{r\rho B^2}\right]_\theta$$

## Mode Shearing Effects of $\omega_*$

Linearly the application of a diamagnetic drift decreases the growth rate of the DTM until some critical value when the system splits into two drifting STM layers.



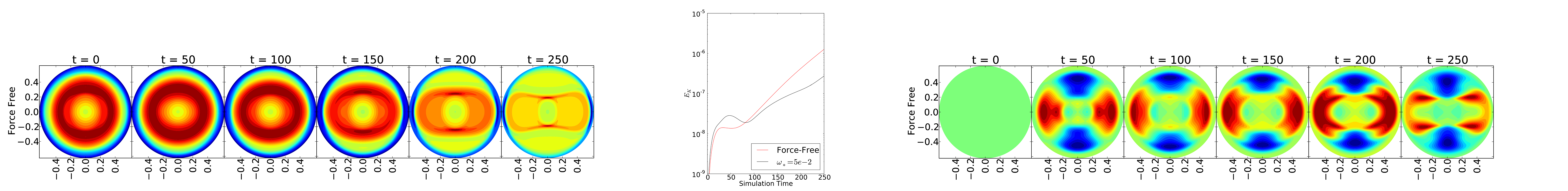

Below the critical decoupling amplitude a difference in drift results in mixing of the real, even (black) and imaginary, odd (red) solutions to the DTM.



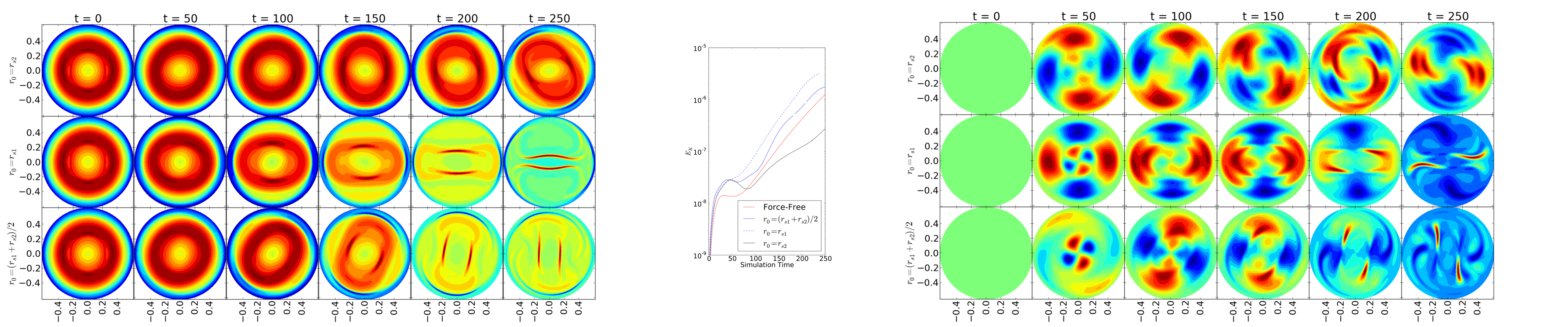
## Nonlinear DTM Simulation Results

Axial Current | Radial Momentum

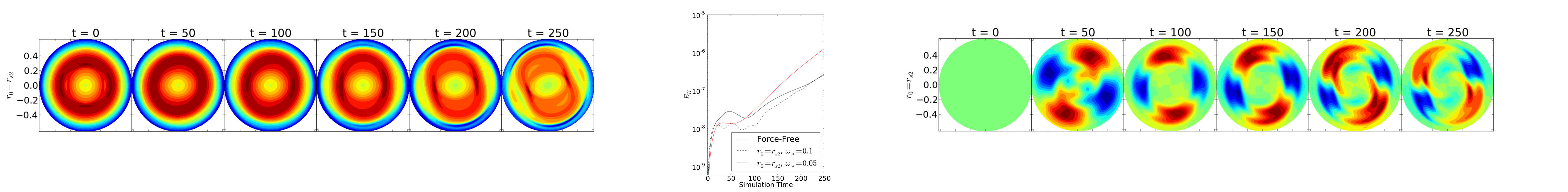
Force Free




$\omega_* = 5e-2$




$\omega_* = 1e-1$




## Setup of Nonlinear Simulations

Based on our previous studies of this system we expect the linear decoupling threshold to lie in the range $0.01 < \Delta\omega_c < 0.05$. We use these bounds as characteristic 'coupled' and 'decoupled' states and examine them non-linearly for different locations of the peak pressure gradient. In addition to localizing the drift at the inner and outer rational surfaces, we consider a case with equal drifts at both locations to eliminate differential rotation effects.

The value of $\omega_*$ at the rational surfaces is determined by several different parameters. For this work we fix $d_i = 0.1$. The pressure profile values $r_0$, $\delta_N$, and $N_b$ are set as follows for the three configurations of interest.

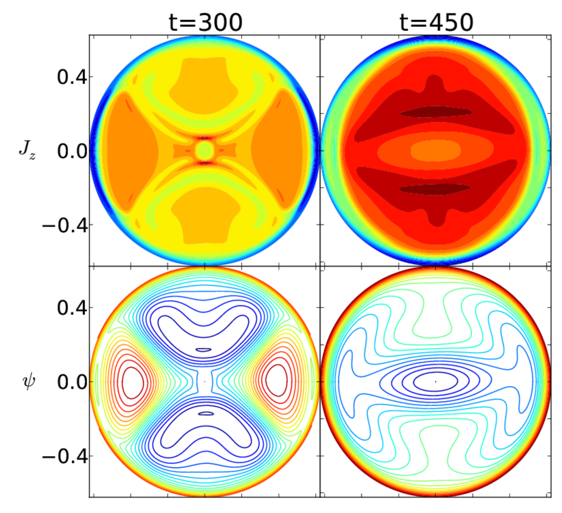
**Equal Drift** $\omega_*$ is equal at both surfaces. $\delta_N = 0.2$ is fixed. The parameters $N_b$ and $r_0$ are set to achieve the desired drifts: $\omega = 0.05$:[$N_b = 0.298$,$r_0 = 0.41$]; $\omega = 0.1$:[$N_b = 0.08$,$r_0 = 0.358$].

**Inner Drift** The pressure gradient is localized around the inner rational surface, with nearly no drift at the outer. The parameters $\delta_N = 0.1$ and $r_0 = r_{s1}$ are fixed. $N_b$ is set to achieve the desired drift: $\omega = 0.05$:$N_b = 0.762$.

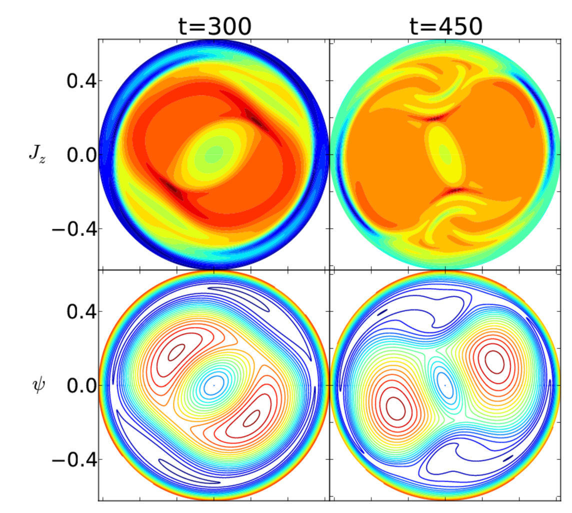
**Outer Drift** The pressure gradient is localized around the outer rational surface, with nearly no drift at the inner. The parameters $\delta_N = 0.1$ and $r_0 = r_{s2}$ are fixed. $N_b$ is set to achieve the desired drifts: $\omega = 0.05$:$N_b = 0.58$; $\omega = 0.01$:$N_b = 0.306$.

## Long Time Behavior

Force-Free




$r_0 = r_{s2}$, $\omega_* = 5e-2$




## Discussion

The stabilizing effects of diamagnetic drifts on the DTM are highly dependent on where the peak of $\omega_*$ is located. Disrupting the interaction between tearing surfaces is a critical component of stabilization. Equal (or nearly equal) strong drifts at both rational surfaces provides some initial suppression of the mode but are overwhelmed nonlinearly. Locating the drift at the outer resonant surface is much more effective. Even in our most stabilized simulations ($\omega_* = 5e-2, 1e-1$ at $r_{s2}$) the mode continues to grow, suggesting that saturation may not be possible.

While many of these observations are specific to this equilibrium, these simulations show both that $\omega_*$ drifts can slow the evolution of the non-linear DTM and that the details of the pressure gradient are critically important. Conducting a similar study for modes other than the $m = 2$, $n = 1$ case shown here would allow for a better understanding of the $\omega_*$ effects on the 'explosive' growth phase of the DTM, which cannot easily be observed in this equilibrium. Broadening the scope of safety factor profiles may also allow a survey of experimental data where locations and strengths of ITB pressure gradients can be correlated with possible DTM driven disruptions.

## References

[1] P. L. Pritchett, et al. "Linear analysis of the double-tearing mode" *Phys. Fluids*, **23** , 1368 (1980)

[2] Y. Ishii, et al. "Nonlinear evolution of double tearing modes" *Phys. Plasmas*, **7** , 447 (2000)

[3] Z. X. Wang, et al. "Fast Resistive Reconnection Regime in the Nonlinear Evolution of the Double Tearing Mode" *Phys. Rev. Lett.* , **99** , 185004 (2007)

[4] A. Bierwage, et al. "Dynamics of resistive double tearing modes with broad linear spectra" *Phys. of Plasmas* , **14** , 022107 (2007)

[5] X. M. Zhao, et al. "Double tearing modes in the presence of internal transport barrier" *Phys. of Plasmas* , **18** , 072506 (2011)

[6] A. Mao, et al. "Eigenmode characteristics of the double-tearing mode in the presence of shear flows" *Phys. of Plasmas* , **20** , 022114 (2013)

[7] B. Rogers, L. Zakharov "Nonlinear $\omega_*$-stabilization of the $m = 1$ mode in tokamaks" *Phys. Plasmas* , **2** , 3420 (1995)