Third Moment Description of the Turbulent Cascade and Intermittency

Jesse T. Coburn[1], Charles W. Smith[1], Miriam A. Forman[2], Bernard J. Vasquez[1], Julia E. Stawarz[3]

[1]University of New Hampshire, [2]SUNY at Stony Brook, [3]University of Colorado - Boulder

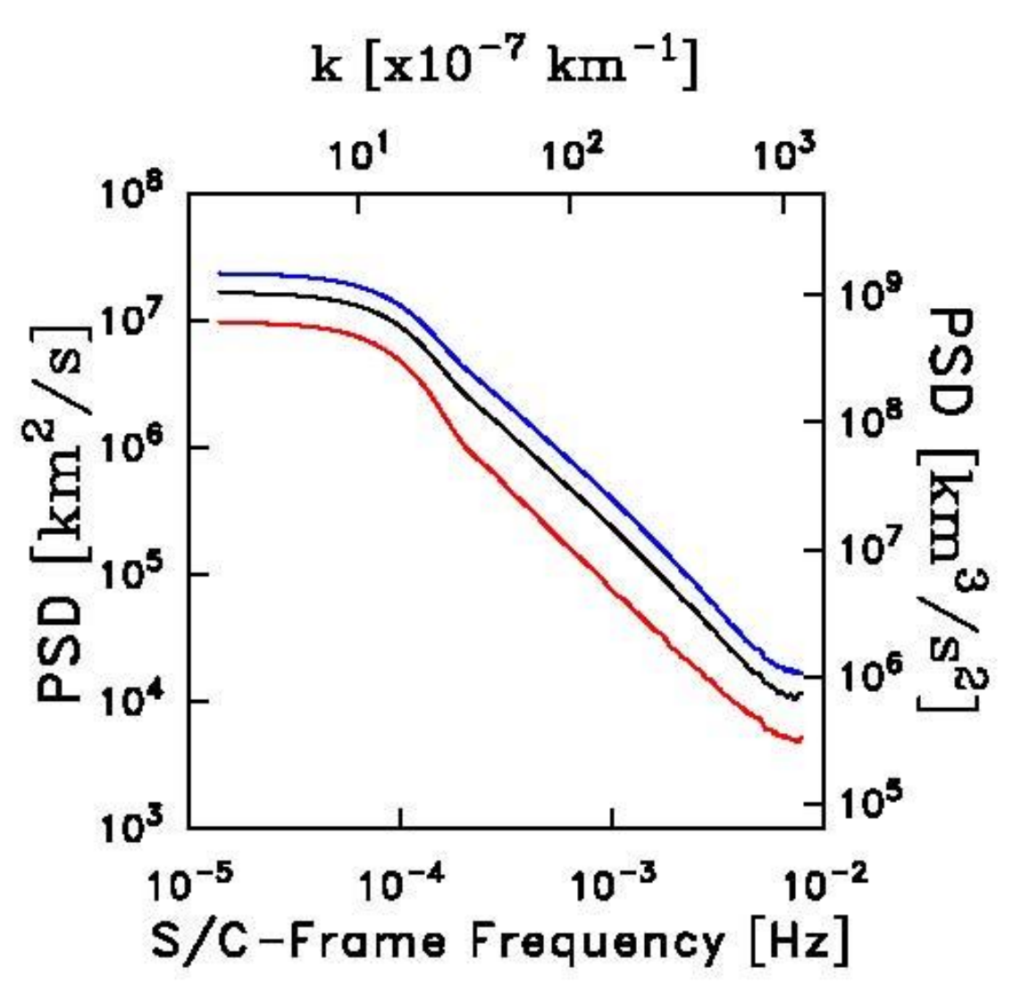
Magnetohydrodynamic Energy Cascade Rate

The energy dissipation rate of an incompressible magnetohydrodynamic [MHD] fluid was derived by Politano & Pouquet [1] following Yaglom [2] by assuming statistical homogeneity and strict spatial independence. Here, we present the isotropic form after assuming the existence of an inertial range and the second moment to be time stationary:

$$\frac{4}{3}\epsilon^{\pm} r_L = -\langle \delta z_L^{\mp}(\mathbf{r}) |\delta z_i^{\pm}(\mathbf{r})|^2 \rangle$$

The scale invariant energy cascade rate is related to a particular third order structure function through the spatial increment.

The Kolmogorov phenomenology predicts the power law and offers a universal description of the small scale structure of turbulence. **Right)** The power spectral density is defined: $E^{tot}(k)$, $E^{out}(k)$, $E^{in}(k)$. The outward fluctuations possess more energy, all three possess a exponent of -5/3 through the inertial range.



Proton Heating Rate in the Solar Wind at 1AU

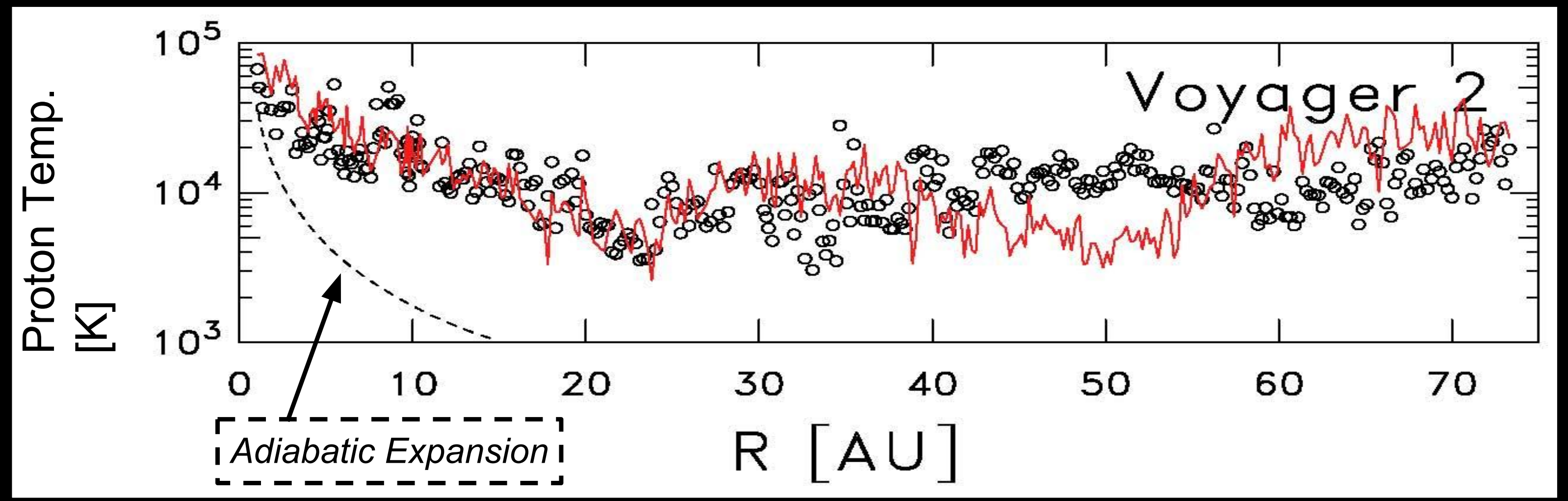
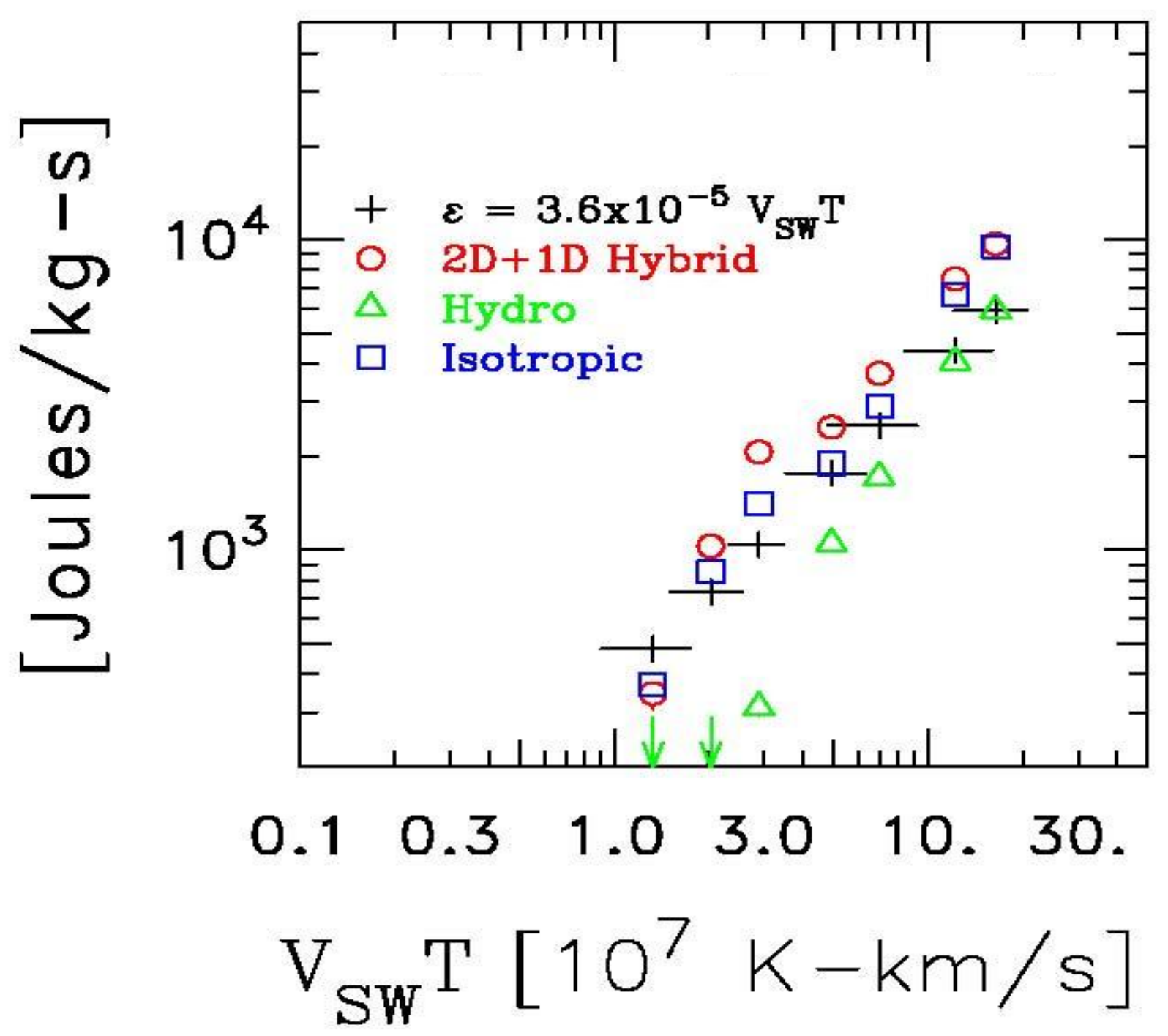


Figure Above) The non-adiabatic radial temperature dependence of solar wind protons (circles) requires in situ heating and pickup ions (red line) [cite].

Figure Right) For a large data set the third moment structure function converges to a scale independent energy cascade rate. The average of the measured cascade rates match the inferred heating rate from the velocity-temperature product. [3,4,5]



Variable Cascade Rates and Intermittency

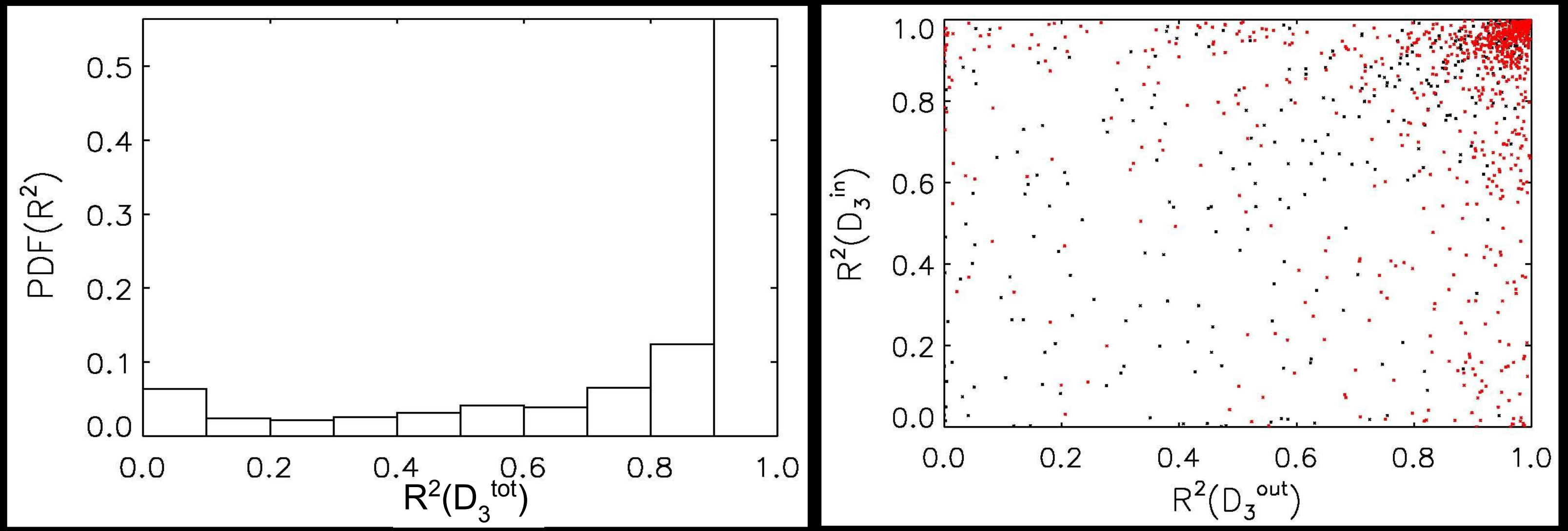
Traditional studies of intermittency account for the non-unique exponents of the structure functions, or measure the kurtosis of the field fluctuations. Our studies measure local dynamics to present a new picture; local inertial range dynamics are well behaved and adjust as large-scale and dissipation scales are not in statistical equilibrium.

The averaging scheme employed in our studies is two-tiered:

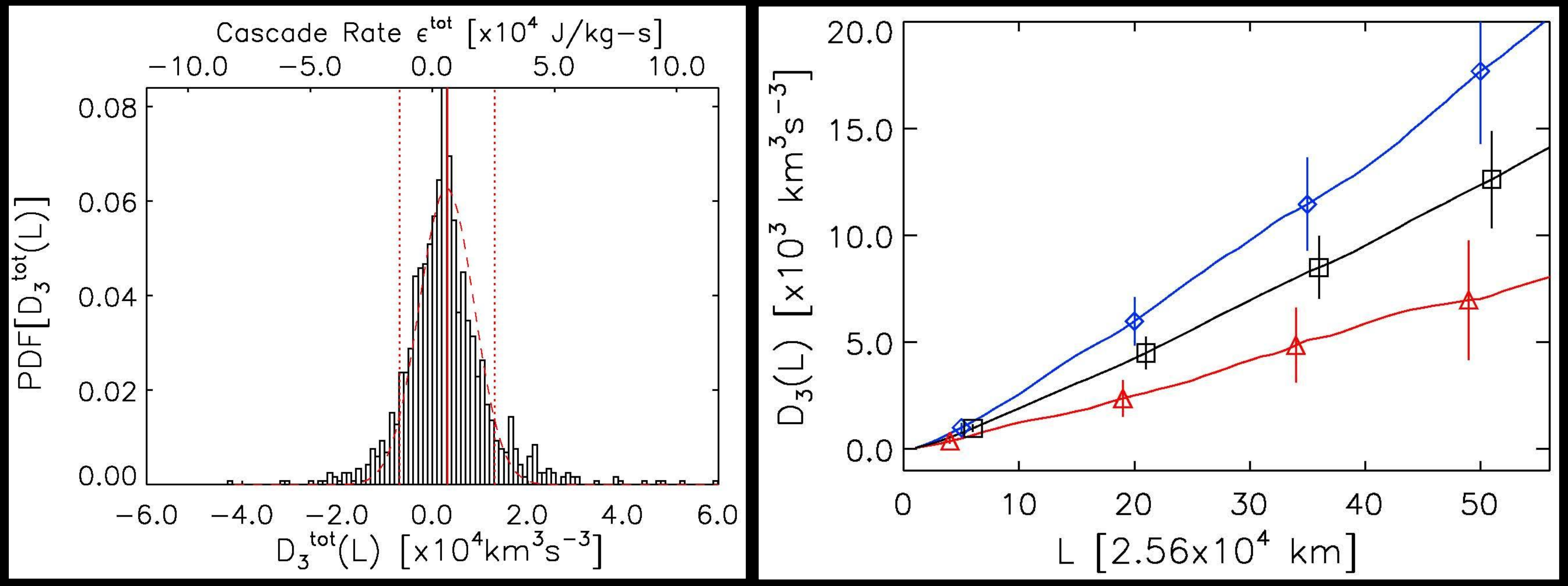
I. Calculate the third moment function for a few correlation lengths [Local Energy Cascade Rate].

II. Interpolate the functions to common lags and then calculate Gaussian statistics [Heating Rate].

Figures Below) The energy range (1200 - 2800 [km^2/s^2]) and normalized cross helicity range (-0.75 - 0.75) was chosen to study the linearity (R^2 = least-squares) of the third moment function. **Left)** More than 70% of the local estimates are linear. **Right)** Red points are $R^2(D_3^{tot}) > 0.7$. Linear D_3^{tot} are often accompanied by linear D_3^{out} and D_3^{in}.



Figures Below) Only $R^2(D_3^{tot}) > 0.7$ are included. **Left)** Many samples produce a well formed distribution and positive average. **Right)** 3rd moment functions estimate a scale independent cascade rate. D_3^{tot}, D_3^{out}, D_3^{in}.



The energy cascade rates are being determined on a local time scale and posses a large degree of variability. Energy transfer in the inertial range displays a large degree of back transfer. The heating rate can be measured from the average of many local energy cascade rates [6].

Local Cascade Rates and Anti-Correlation

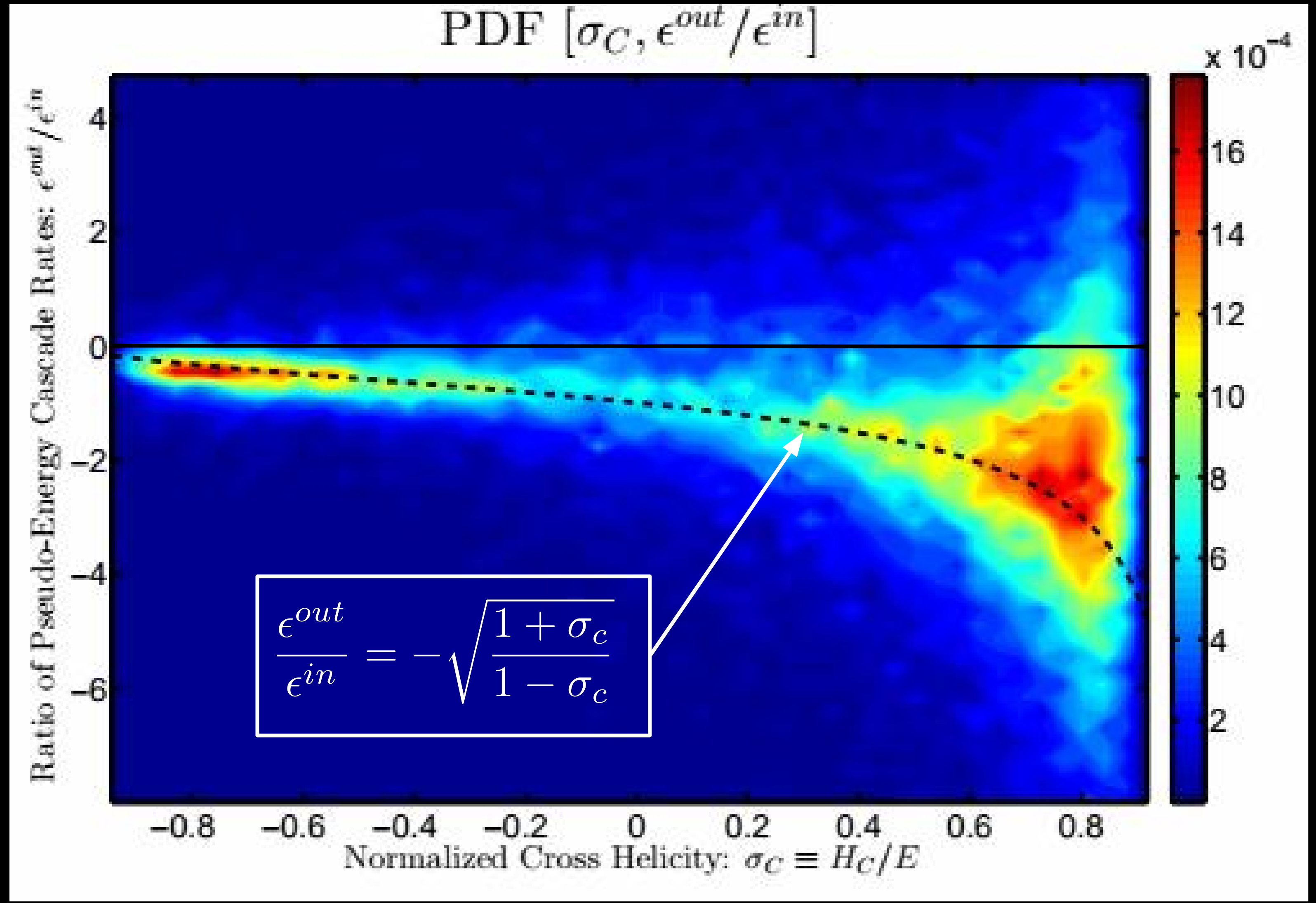
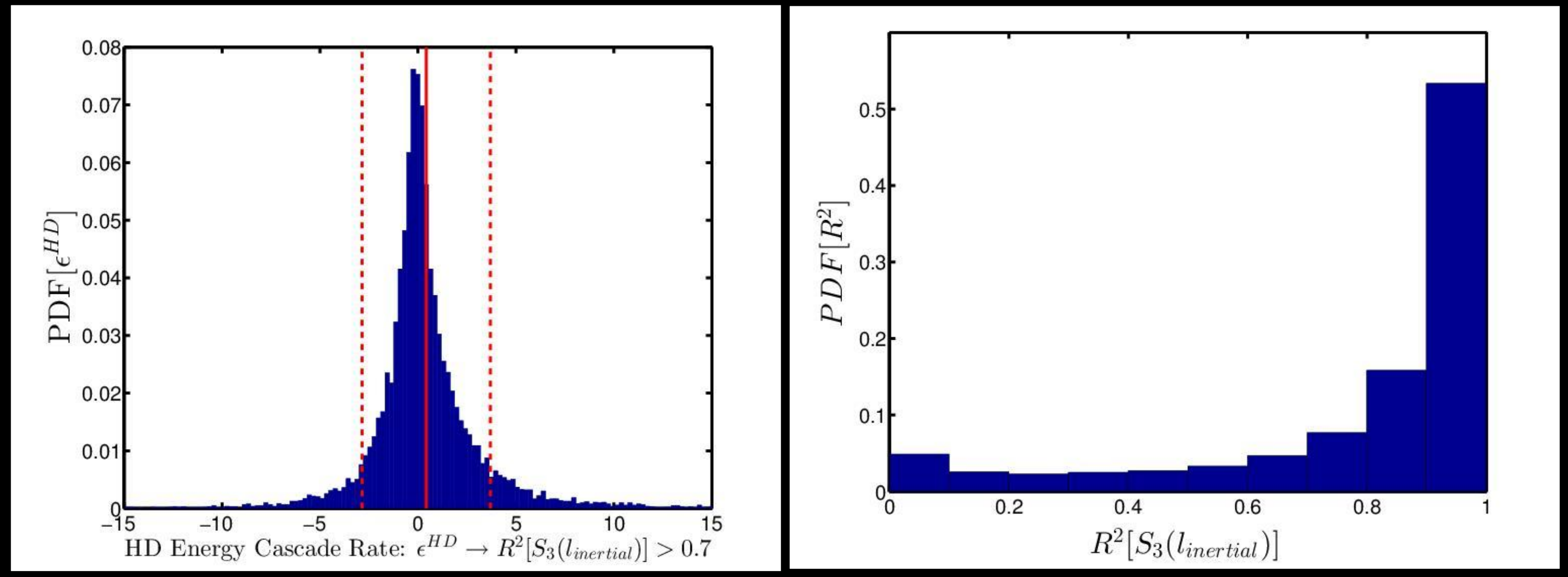


Figure Above) The ability of the third moment technique to estimate the local energy cascade rate can then be employed to study various decay processes and regions of the solar wind [7].

Hydrodynamic Cascade and Intermittency

The measurement [Modane Wind Tunnel] of the hydrodynamic energy cascade rate produces very similar results to the solar wind. The inertial range bounds are not in statistical equilibrium on similar time-scales. This creates a very dynamic picture of the energy transfer in the inertial range bound by local quantities [8].



References and Acknowledgements

[1] Politano & Pouquet, P. Rev., 1998a
[2] Yaglom, DAN SSSR, 1949
[3] Smith et al., ApJ, 2006
[3] Vasquez et al., JGR, 2007
[4] Lamarche et al., JGR, 2014
[5] Stawarz et al., ApJ, 2009
[6] Coburn et al. ApJ, 2012
[7] Coburn et al. Phil. Trans. 2015
[8] Coburn et al. (In Press)

Background Figure: SolarDynamicObservatory/NASA