



A Scaling Law for Fractal Contact and Interlocking

University of
New Hampshire



Mona Monsef¹ and Yanning Li*
Department of Mechanical Engineering, University of New Hampshire, Durham, NH 03824
1: mkm10115@wildcats.unh.edu, *: yanning.li@unh.edu



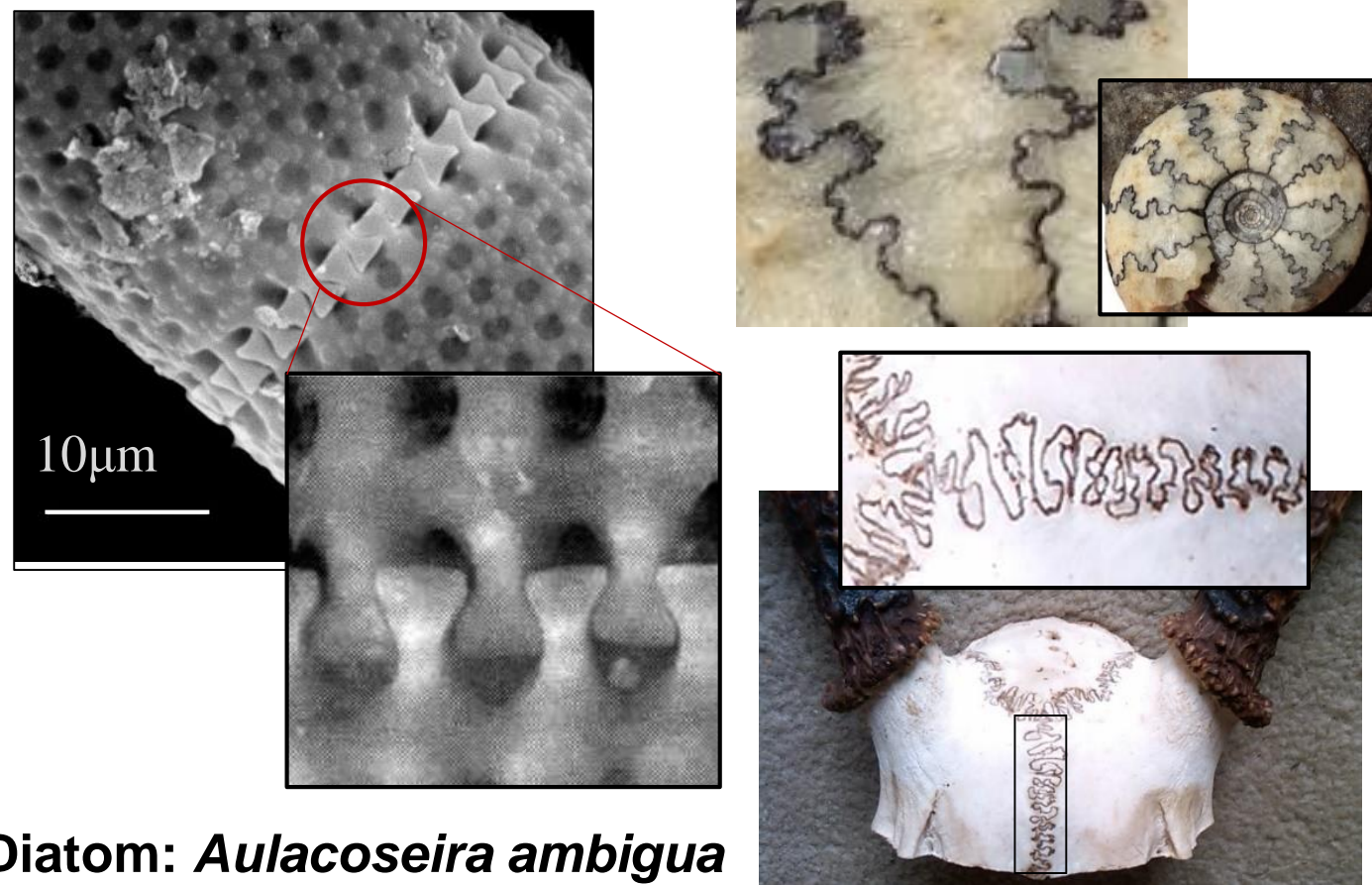
ABSTRACT

Inspired by topological interlocking in nature, Koch fractal interlocks were designed and fabricated via 3D-printing. Integrated theoretical-numerical-experimental models were developed to evaluate the role of the order of hierarchy N and geometric imperfections.

Motivation and Objective

Topological interlocking in nature

Ammonite: *Craspedites nodiger*



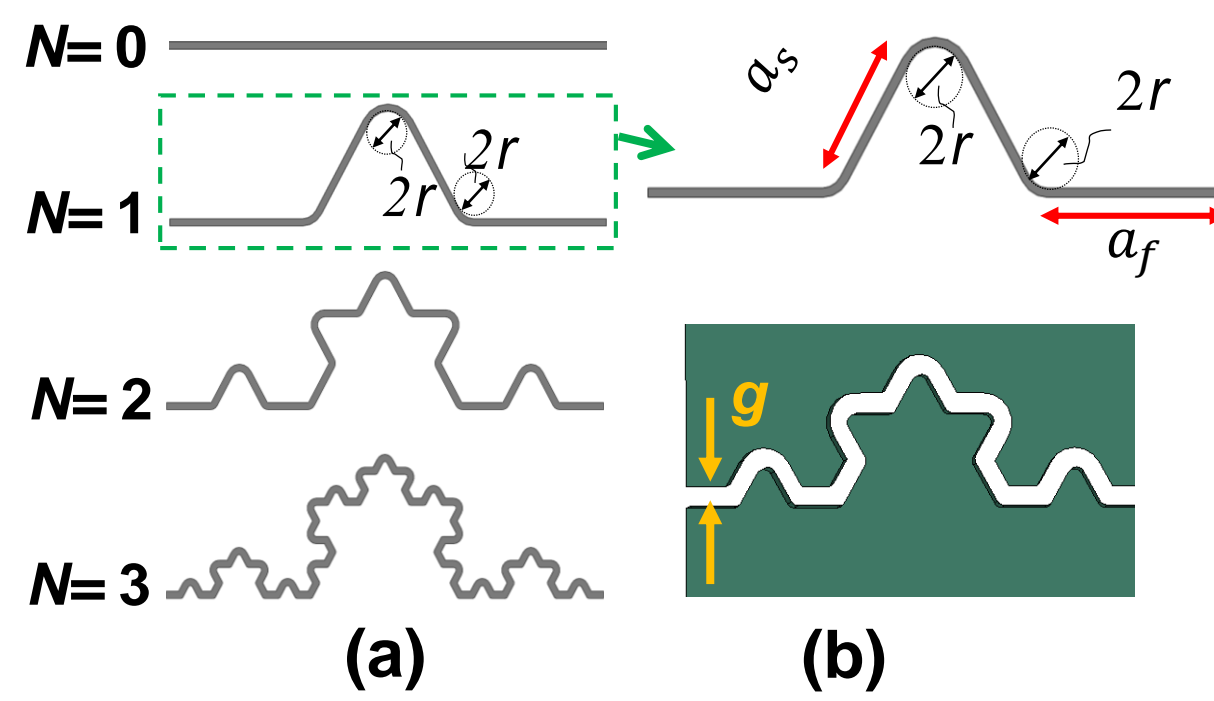
Diatom: *Aulacoseira ambigua*

Deer skull

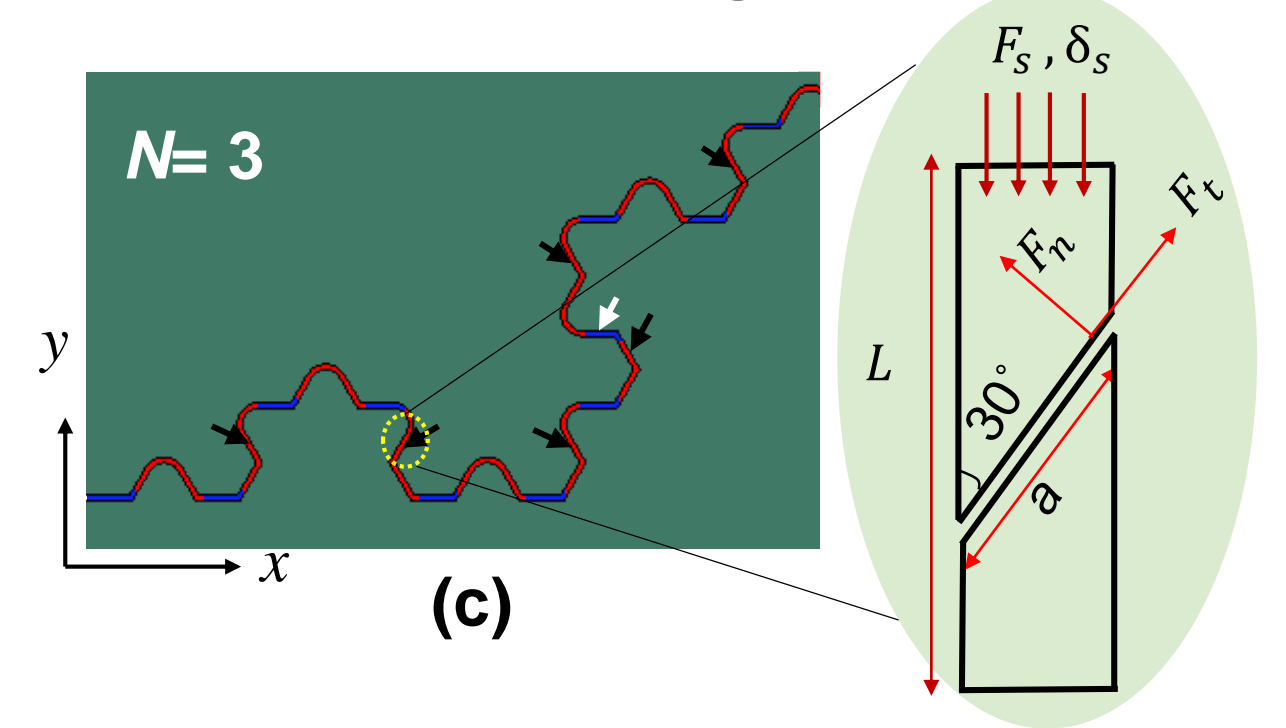
The goal of this research is to evaluate the mechanical performance of an innovative design of fractal contact and interlocking.

Design and Theoretical Model

Self-similar Koch fractal interlocking
Geometry of the models is defined by g , r , N and a_N

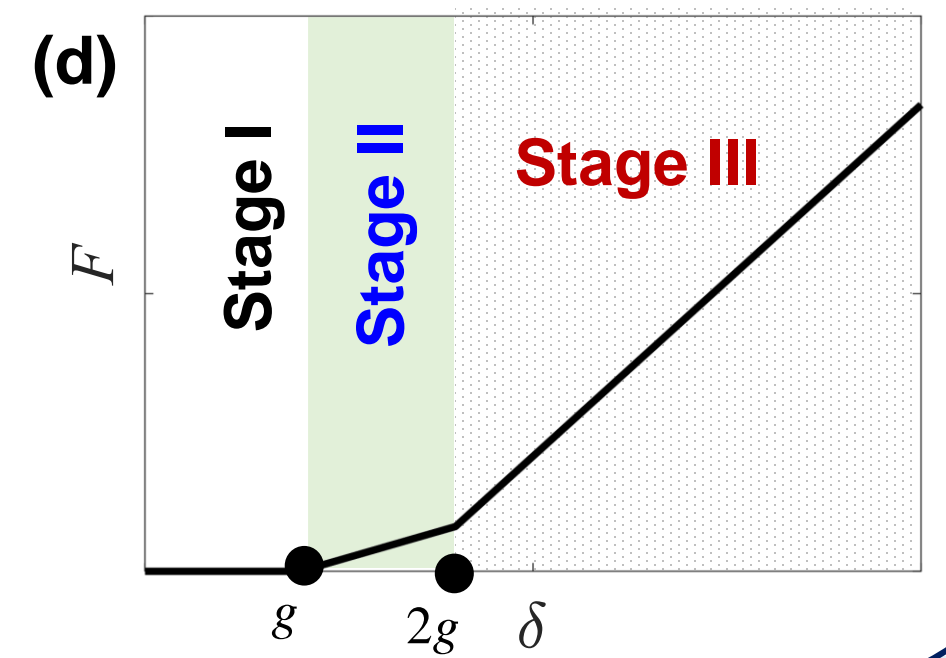


Non-Hertzian, Conformal Contact of Flat and Slant segments



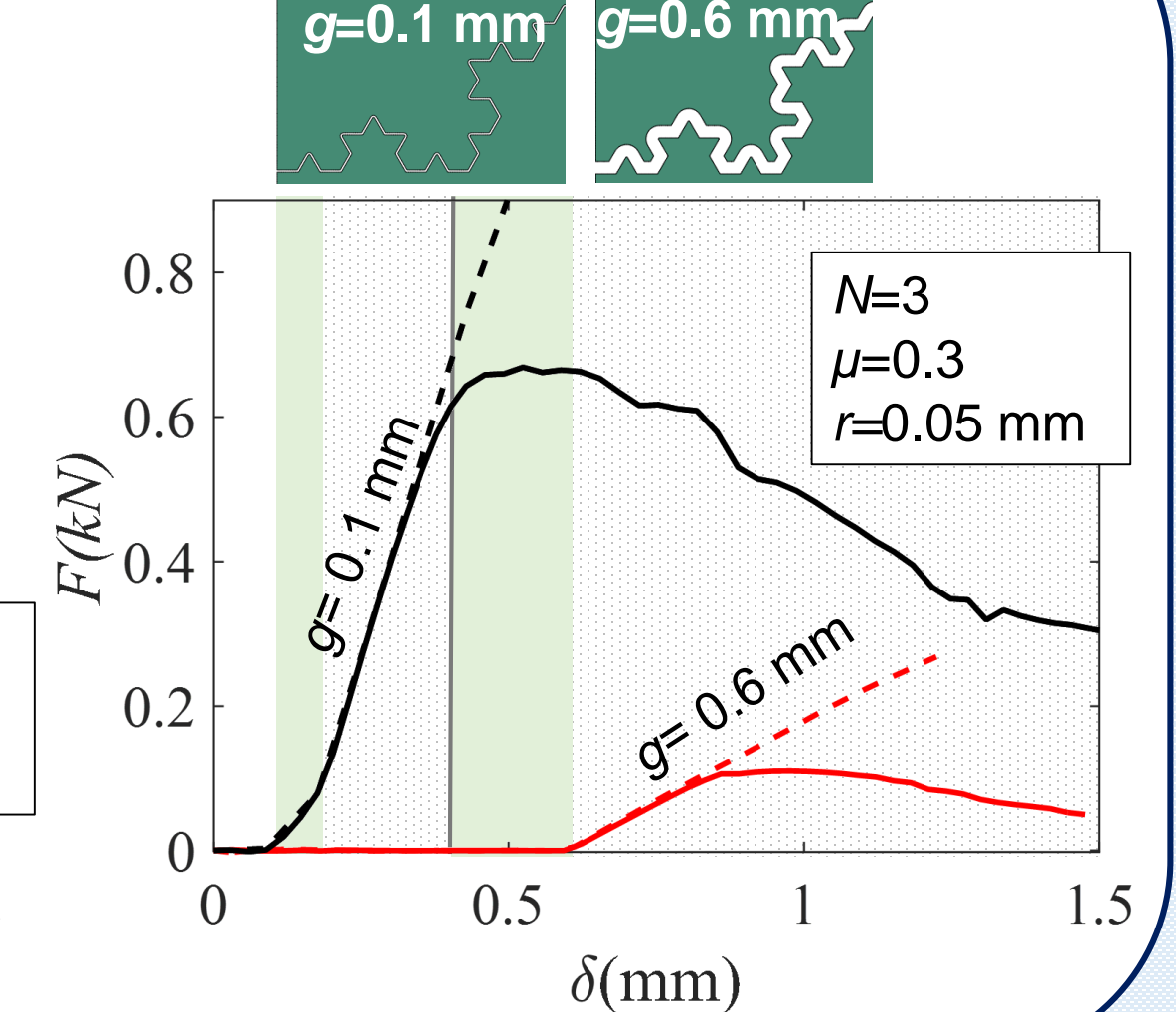
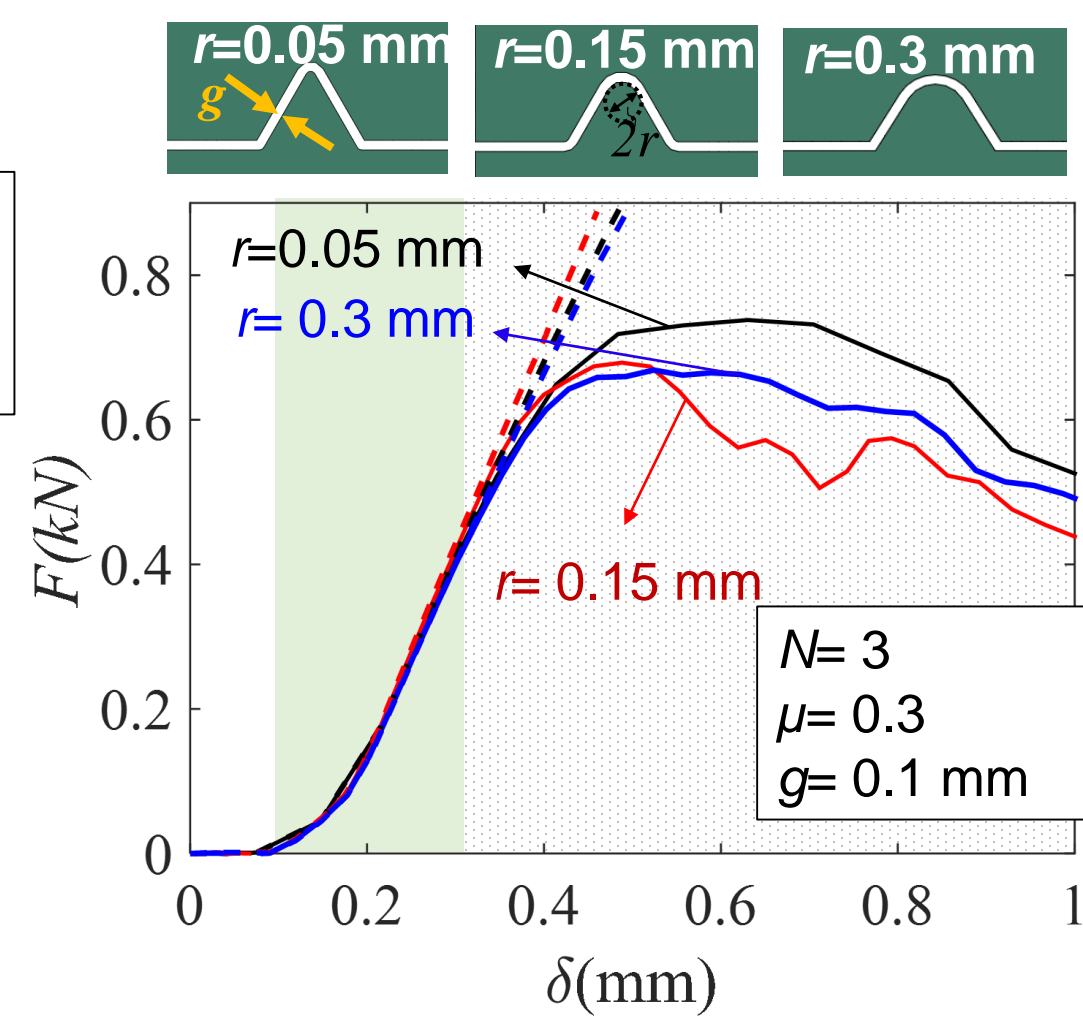
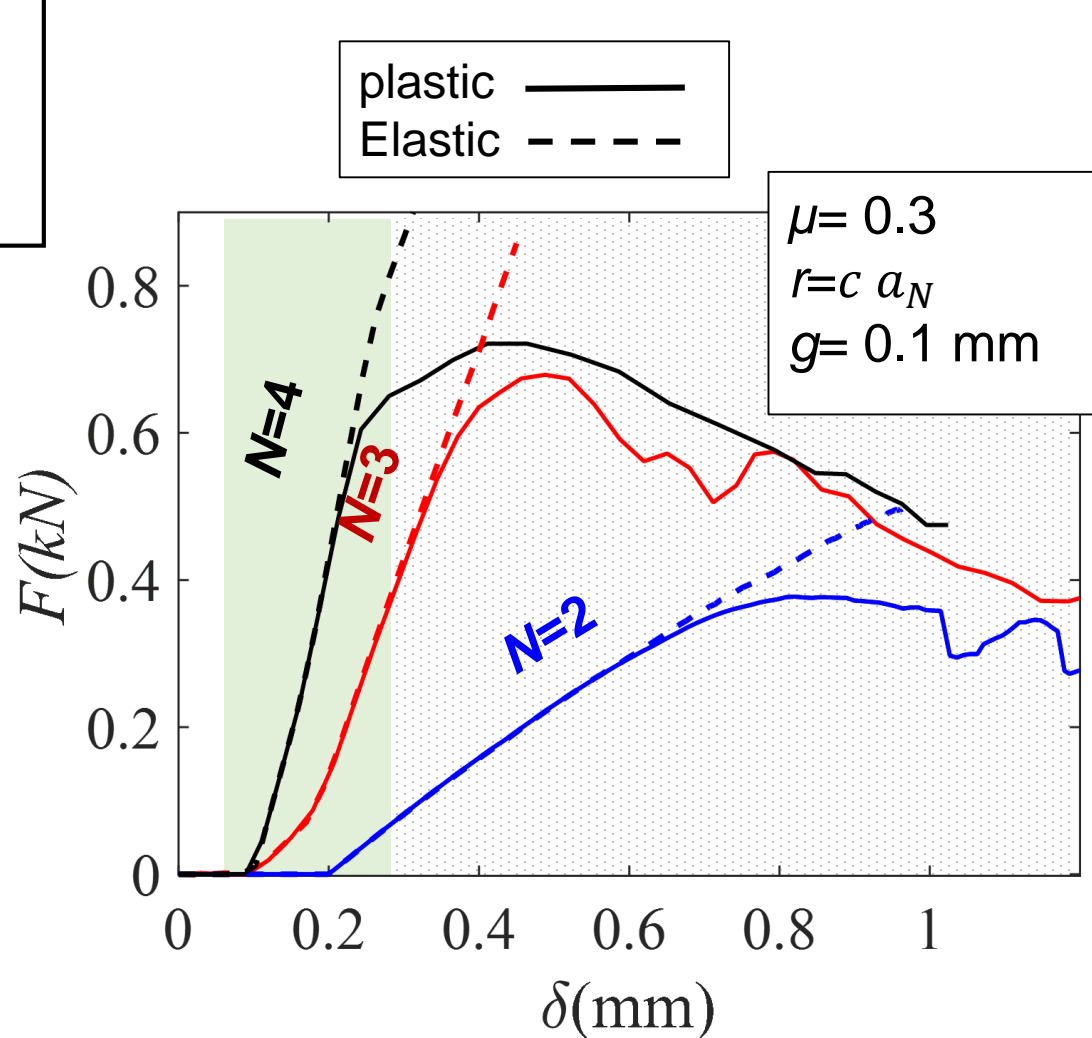
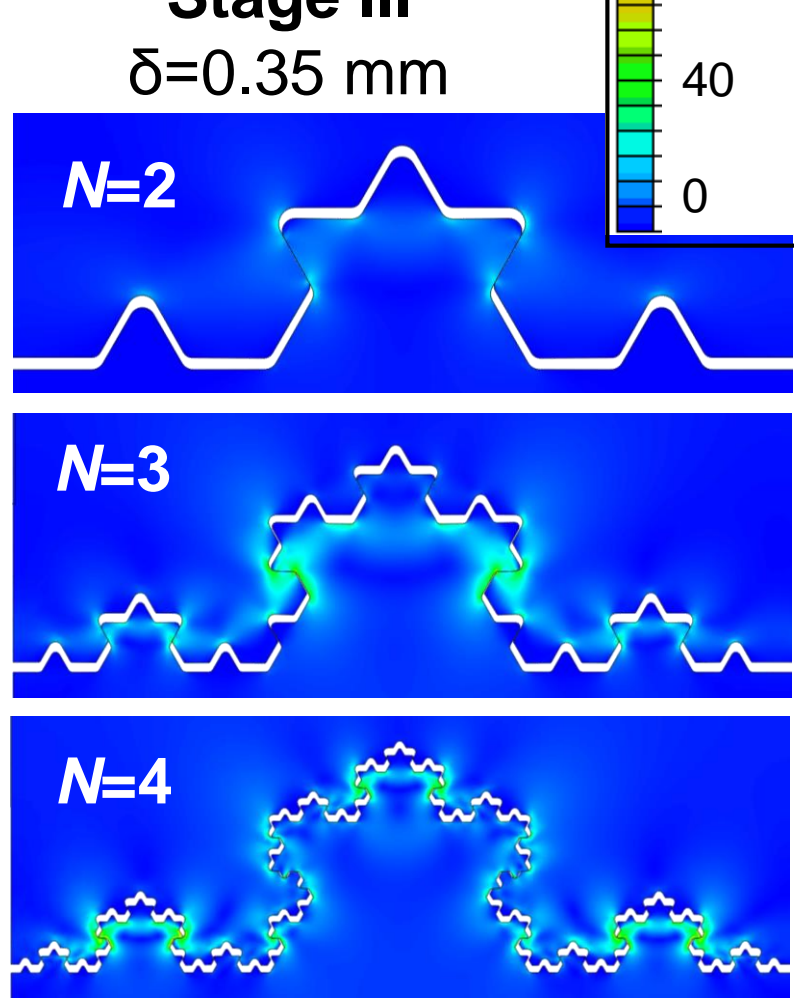
Deformation mechanism

Stage I: $F(N) = 0$; $\delta < g$
Stage II: $F(N) = f_n a_n n_f^c [N]$; $g < \delta < 2g$
Stage III: $F(N) = f_s a_s n_s^c [N] + f_n a_n n_f^c [N]$; $\delta > 2g$



Finite Element Results

Von-Mises stress Stage III
 $\delta = 0.35$ mm



Experimental Results

Experimental method:

- Uniaxial tension on Zwick machine
- Displacement rate of 0.024 mm/min
- Digital Image Correlation (DIC)

Observations:

- When N increases, stiffness, strength and toughness increases.
- Adding a soft layer can significantly increase stiffness, strength and toughness for all N s.

$\epsilon_{yy} = 0.05$

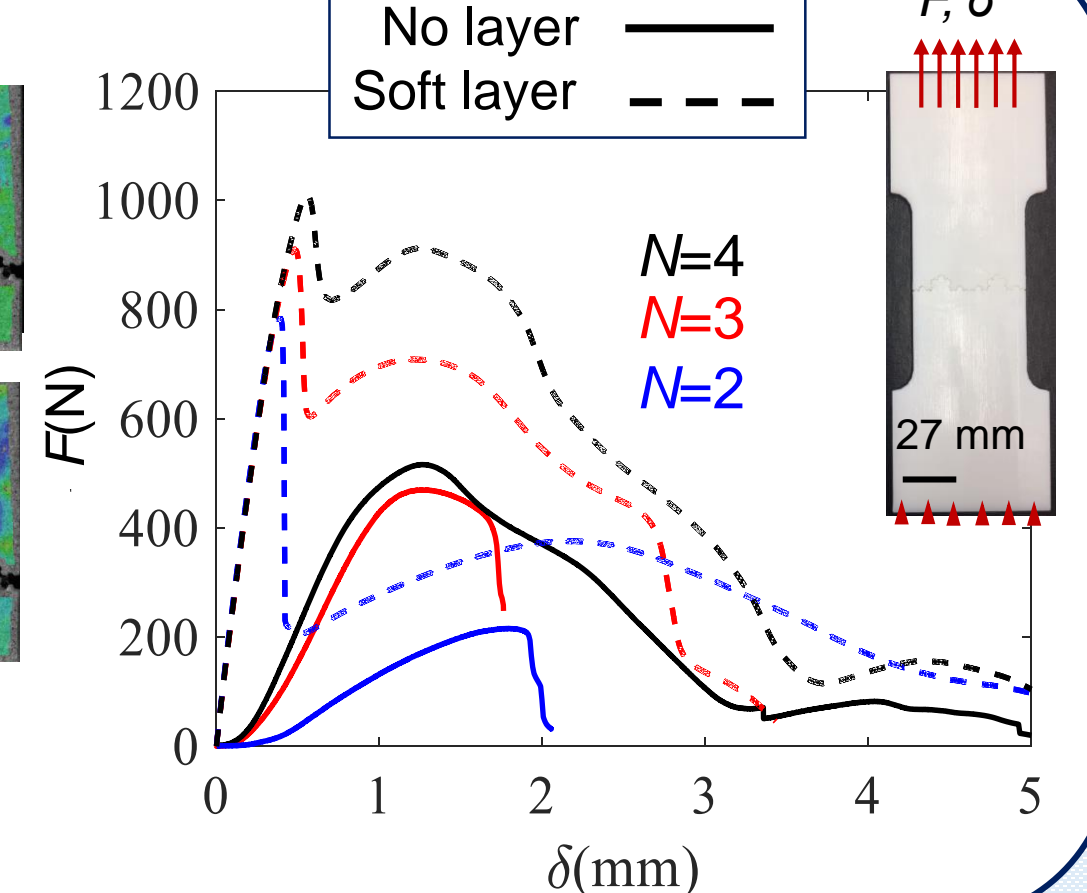
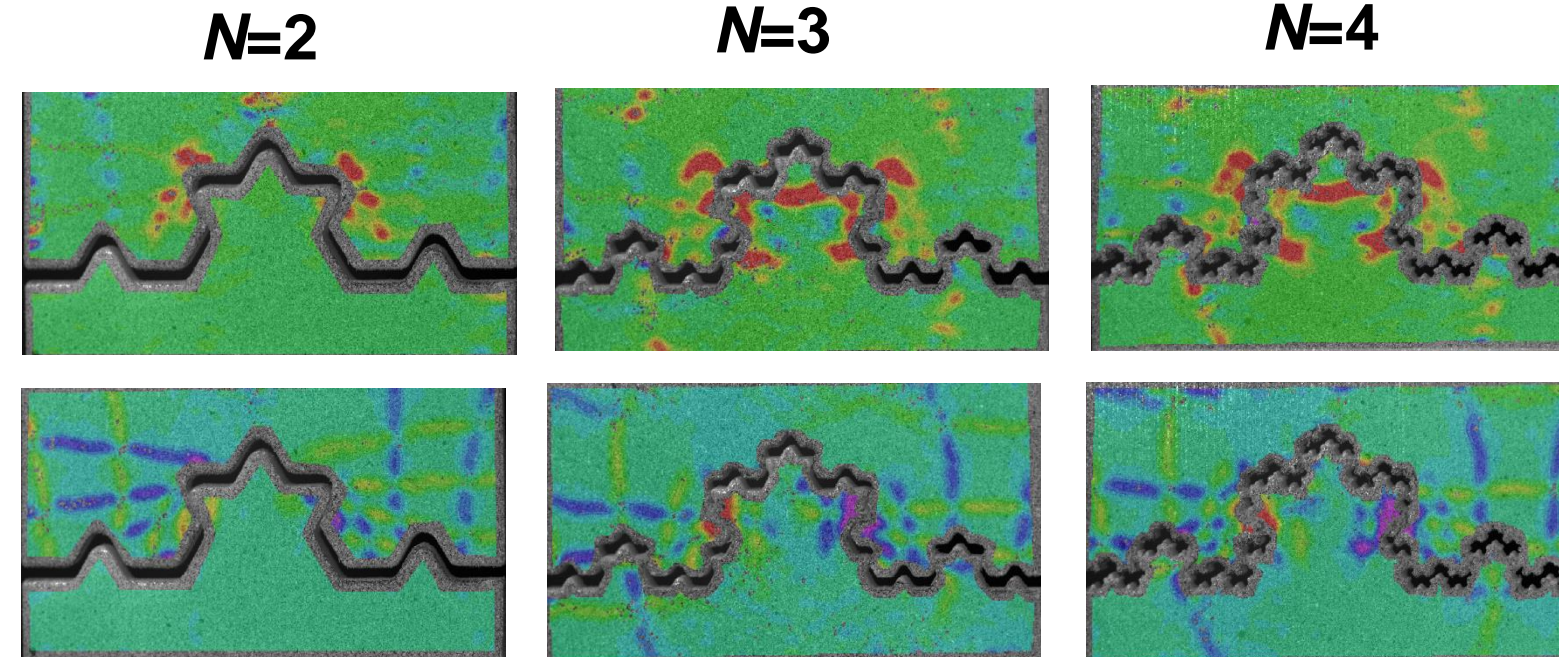
0.017

-0.020

$\epsilon_{xy} = 0.05$

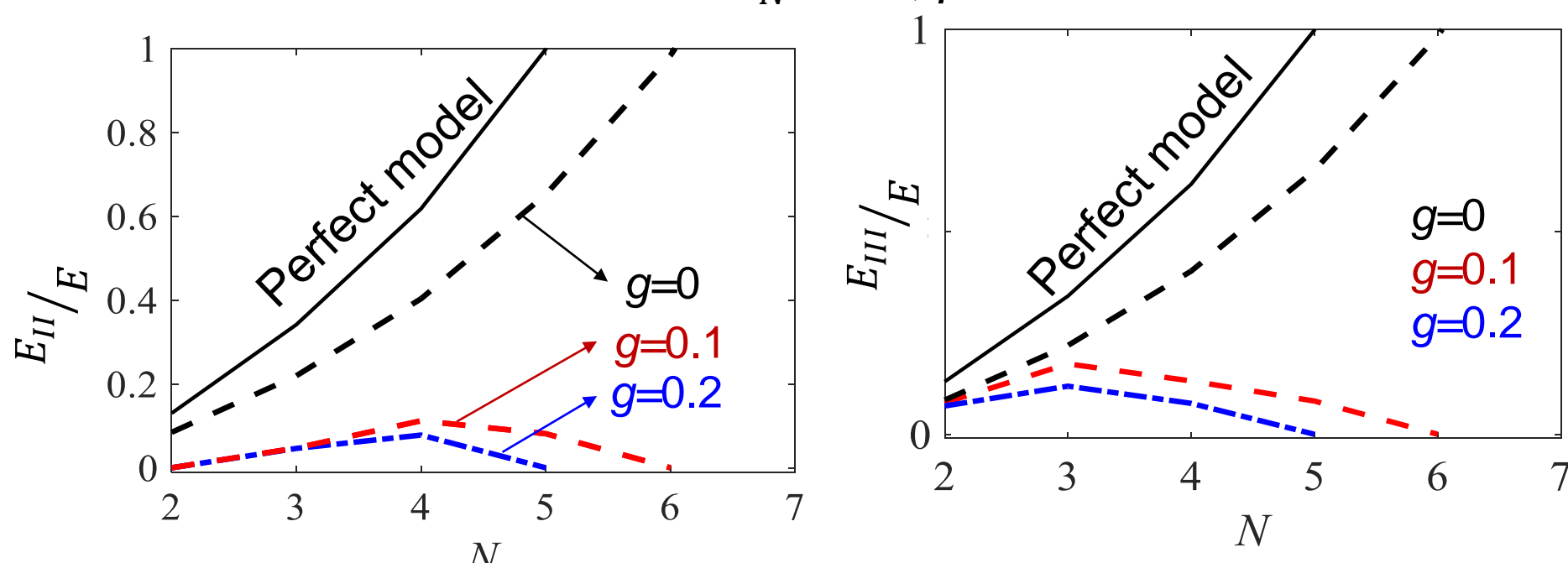
0.03

-0.024



A Scaling Law

$r = 0.15 a_N$ mm, $\mu = 0.1$



The ideal model with no imperfection (i.e. $g=0$ and $r=0$) provides the upper bound of the stiffness of the fractal contact and interlocking

Due to geometric imperfection g and r , an optimal N exists for maximum stiffness.

Conclusion

- Koch fractal geometry can significantly improve energy dissipation, stiffness and strength of topological interlocking.
- The mechanical properties of Koch fractal contact and interlocking are sensitive to geometric imperfections.

References

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Acknowledgement

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