Instability Thresholds for Oblique Alfvén/Cyclotron Modes in the Presence of an Alpha Particle Beam

Daniel Verscharen and Benjamin Chandran

Space Science Center, University of New Hampshire, Durham, NH 03824, USA (daniel.verscharen@unh.edu)

Introduction

Alpha particles are a dynamically important species in the fast solar wind. They often show a relative drift with respect to the protons along the direction of the magnetic field. Statistical analyses indicate that this drift is limited to a speed of order the local proton Alfvén speed v_A (Fig. 1), and microphysical instabilities are believed to regulate the drift.

Solutions to the kinetic dispersion relation of a hot plasma show that ion beams can excite different modes with typically lower thresholds at oblique angles of propagation compared to the parallel case (see Fig. 2).

We explain the different instability thresholds and identify the responsible resonances in the framework of quasilinear theory.

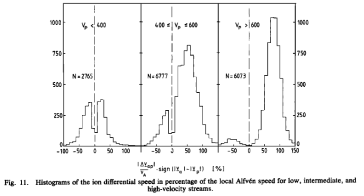

Fig. 1: Histograms of alpha drifts observed with Helios. From Marsch et al. (1982).

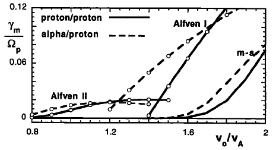

Fig. 2: Beam-instability growth rates in linear theory. From Gary et al. (2000).

Dispersion relation, polarization, and wave energy

We determine the wave frequencies ω at given wavevectors *k* with the dispersion relation of a cold plasma:

$$\frac{\boldsymbol{k}c}{\omega} \times \left(\frac{\boldsymbol{k}c}{\omega} \times \boldsymbol{E}_k\right) + \epsilon \boldsymbol{E}_k \equiv \mathcal{D}\boldsymbol{E}_k = 0.$$

Solutions are calculated using a combined Laguerre-Newton method with a realistic electron-to-proton mass ratio (see results in Fig. 3).

The polarization of the waves can be directly determined from the first two lines of the dispersion relation, where we introduce the definitions

$$E_{k,\mathrm{r}} \equiv (E_{kx} - iE_{ky})/\sqrt{2}, \qquad E_{k,\mathrm{l}} \equiv (E_{kx} + iE_{ky})/\sqrt{2}$$

The wave energy is given by

$$W = \frac{1}{8\pi}\left[\boldsymbol{B}_k^* \cdot \boldsymbol{B}_k + \boldsymbol{E}_k^* \cdot \frac{\partial}{\partial\omega}(\omega\epsilon_{\mathrm{h}})\boldsymbol{E}_k\right]\Bigg|_{\omega=\omega_{\mathrm{r}}}$$

The wave energy can be negative. In that case, resonant damping leads to an increase of the wave amplitude. The C⁻ wave has negative energy, also when it merges with other branches.

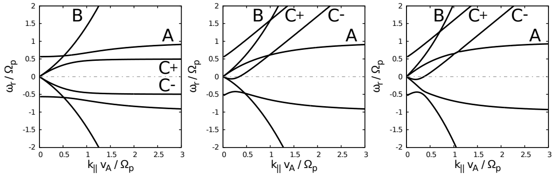

Fig. 3: Cold-plasma dispersion relation. A: proton-cyclotron wave, B: fast magnetosonic/whistler wave, C: alpha-cyclotron wave (+/-: forward/ backward). Left: $U_i = 0$ and $\theta = 0$, middle: $U_i = 1.1v_A$ and $\theta = 0$, right: $U_i = 1.1v_A$ and $\theta = 45°$.

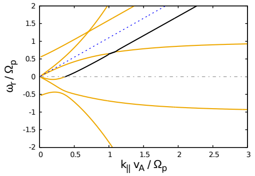

Fig. 4: Wave-energy state. orange: positive, black: negative.

Quasilinear theory and instabilities

We assume that the resonant interaction between the alpha particles and the waves can be described as quasilinear diffusion in velocity space:

$$\frac{\partial f_j}{\partial t} = \lim_{V\to\infty}\sum_{n=-\infty}^{+\infty}\frac{q_j^2}{8\pi^2 m_j^2}\int\frac{1}{Vv_\perp}Gv_\perp\delta(\omega_{k\mathrm{r}} - k_\parallel v_\parallel - n\Omega_j)|\psi_{n,k}|^2 Gf_j \mathrm{d}^3k$$

with

$$G \equiv \left(1 - \frac{k_\parallel v_\parallel}{\omega_{k\mathrm{r}}}\right)\frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega_{k\mathrm{r}}}\frac{\partial}{\partial v_\parallel},$$

$$\psi_{n,k} \equiv \frac{1}{\sqrt{2}}\left[E_{k,\mathrm{r}}e^{i\phi}J_{n+1}(\sigma_j) + E_{k,\mathrm{l}}e^{-i\phi}J_{n-1}(\sigma_j)\right] + \frac{v_\parallel}{v_\perp}E_{kz}J_n(\sigma_j).$$

Particles diffuse in velocity space along paths of constant energy in the wave frame from high particle concentration to low particle concentration (Fig. 5). Along the diffusion paths, the resonant particles can gain (wave damping) or lose (wave instability) kinetic energy. The conditions for instability are:

1. Parallel phase velocity $\omega_r/k_{||}$ of the waves between zero and beam velocity U_i.
2. Resonance condition $\omega_r = k_{||}v_{||} + n\Omega_\alpha$ is fulfilled.
3. Resonance with $n = +1$ requires non-zero $E_{k,l}$, $n = -1$ requires non-zero $E_{k,r}$, and $n = 0$ requires non-zero E_{kz} or non-zero $k_\perp$.
4. No proton damping in the resonant wavenumber range.

For waves with negative energy (C⁻), these criteria correspond to wave damping.

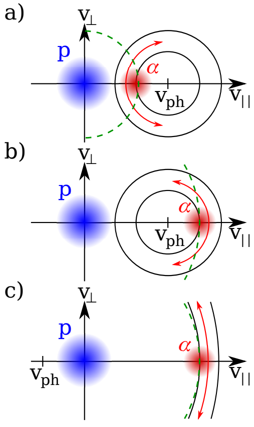

Fig. 5: Diffusion paths (arrows) in *v*-space depending on beam speed and phase speed of the waves. Green circles are iso-contours of kinetic energy. Only case b) is unstable.

Results

The polarization is here shown in color coding (*+1*: purely $E_{k,r}$, *-1*: purely $E_{k,l}$).

Fig. 6 shows the case of parallel propagation. The only possible drift instability in this situation is due to the *n = -1* resonance with the magnetosonic mode. It has lower thresholds at higher thermal speeds of the alpha particles.

Fig. 7 shows the oblique case. Now the proton-cyclotron mode has finite $E_{k,r}$ and can resonate with *n = -1* at lower drift speeds. The magnetosonic and the proton-cyclotron mode can also be unstable due to the Landau resonance at oblique propagation.

Alfvén I: Cyclotron-resonance overcomes proton damping for $v_{||} > 1.1v_A$ for low-β plasma.

Alfvén II: Landau-resonance overcomes proton damping for $v_{||} > 0.8v_A$ (high T_e required to lower the effect of Landau damping by electrons).

Magnetosonic: Cyclotron-resonance triggers instability at higher $v_{||}$ (due to thermal or beam speed).

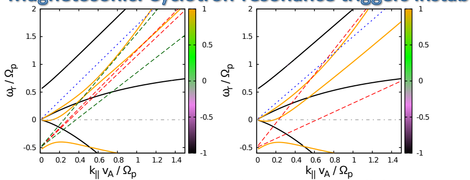

Fig. 6: Dispersion relation and polarization for parallel propagation. Left: $U_i = 1.7v_A$, right: $U_i = 1.5v_A$.

The dashed lines represent the resonance conditions for different parallel thermal speeds of the alpha particles. Maximum parallel speeds correspond to $0.05v_A$ (red dashed lines in Fig. 6 left and Fig. 7), $0.35v_A$ (green dashed lines in Fig. 6 left), $0.7v_A$ (Fig. 6 right).

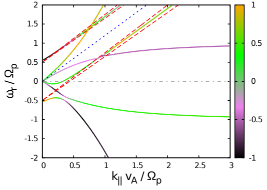

Fig. 7: Oblique propagation, $U_i = 1.1v_A$.

References

S. P. Gary, L. Yin, D. Winske, and D. B. Reisenfeld: *Electromagnetic alpha/proton instabilities in the solar wind*, Geophys. Res. Lett. 27, 1355, 2000

E. Marsch, H. Rosenbauer, R. Schwenn, K.-H. Mühlhäuser, and F. M. Neubauer: *Solar wind helium ions – Observations of the HELIOS space probes between 0.3 and 1 AU*, J. Geophys. Res. 87, 35, 1982

D. Verscharen and B. D. G. Chandran: *The Dispersion Relations and Instability Thresholds of Oblique Plasma Modes in the Presence of an Ion Beam*, Astrophys. J., submitted, 2012

University of New Hampshire