Simulations of Double Tearing Modes in Cartesian and Cylindrical Geometries using the MRC Code

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Introduction

Non-monotonic safety factor profiles show promise for improved confinement but may be susceptible to double tearing mode (DTM) instabilities. Two (or more) nearby rational surfaces may couple and interact both linearly and nonlinearly[1].
Many double-tearing mode studies are performed in a cartesian slab geometry. In force-free equilibria the DTM is strongly localized near the tearing surfaces, which suggests the equivalence of cylindrical and slab geometries. In case of force-balanced equilibria and nonlinear behavior, however, it is unclear if this equivalence holds. We present cylindrical simulation results of $m = 2$, $n = 1$ DTMs in the presence of an ITB-like pressure gradient which exhibit behavior similar to the ideally unstable $m = 1$ kink tearing mode. This behavior is not present in equivalent cartesian simulations.

Equilibria

Cylindrical We use the non-monotic safety factor profile from Ref [2]:

$$q(r) = q_0 F_1(r)\{1 + (r/r_0)^{2w(r)}\}^{1/w(r)} \qquad r_0 = r_A|[m/(nq_0)]^{w(r_A)} - 1|^{-1/[2w(r_A)]}$$

$$w(r) = w_0 + w_1 r^2 \qquad F_1(r) = 1 + f_1 \exp\{-[(r - r_{11})/r_{12}]^2\}$$

with the constant values: $r_A = 0.655$, $w_0 = 3.8824$, $w_1 = 0$, $f_1 = -0.238$, $r_{11} = 0.4286$, $r_{12} = 0.304$, $m = 2$, $n = 1$. q_0 may be varied near 2.5 to change the separation D between two $q = 2$ surfaces. Assuming $B_{z0} = R_{major} = 10$ we find the in-plane field B_θ. For this work we fix $q_0 = 2.5$, giving $D \approx 0.26$. Density profiles are of the form [3]:

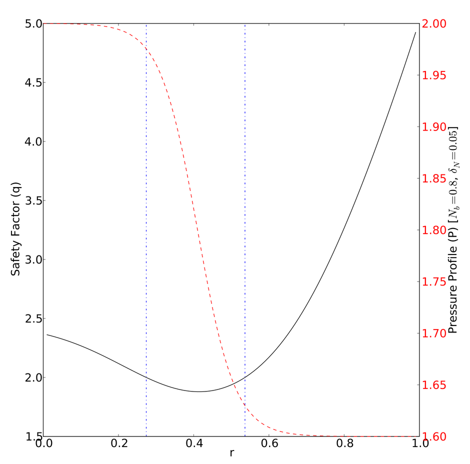
$$N(r) = N_0\{1 - (1 - N_b\frac{\tanh(r_0/\delta_N) + \tanh[(r - r_0)/\delta_N]}{\tanh(r_0/\delta_N) + \tanh[(1 - r_0)\delta_N]}\}$$

Where $N_0 = 1$ and r_0 is chosen equidistant from the two $q = 2$ surfaces. Temperature $T = 1$ is uniform.

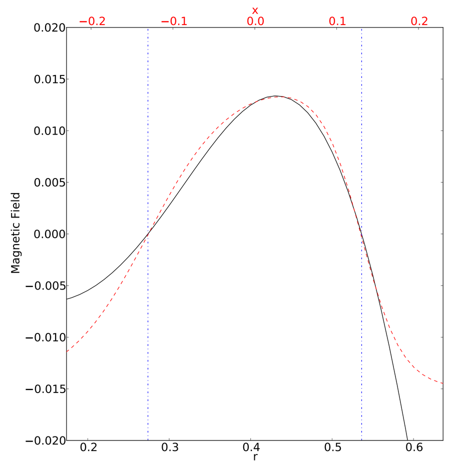
Cartesian To approximate the helical field we choose:

$$B_y(x) = -B_y^0(\tanh[(x - x_0^+)/\lambda^+] - \tanh[(x - x_0^-)/\lambda^-] + 1)$$

where $B_y^0 = 0.015$, $x_0^\pm = \pm D/2 = \pm 0.13$, $\lambda^+ = 0.05$, $\lambda^- = 0.1$. The same density profile is used with $r \to x$, $r_0 = 0$, and the asymptotic guide field $B_{z0} = 10$ is varied to maintain pressure balance.

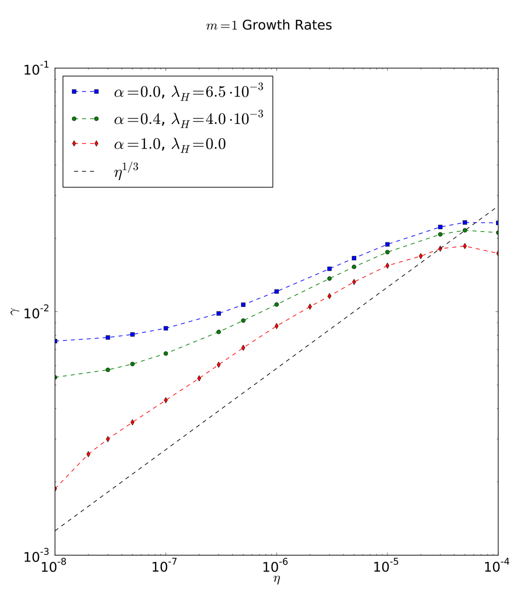


Cylindrical Equilibrium

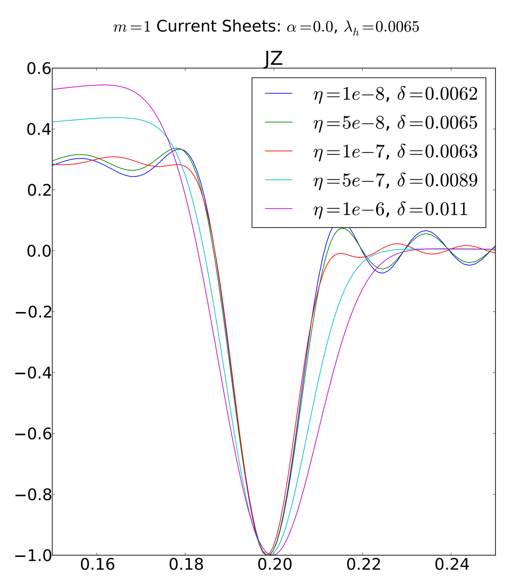


Cartesian Translation: Helical, B_y

$m = 1$ Ideally Unstable Kink-Tearing

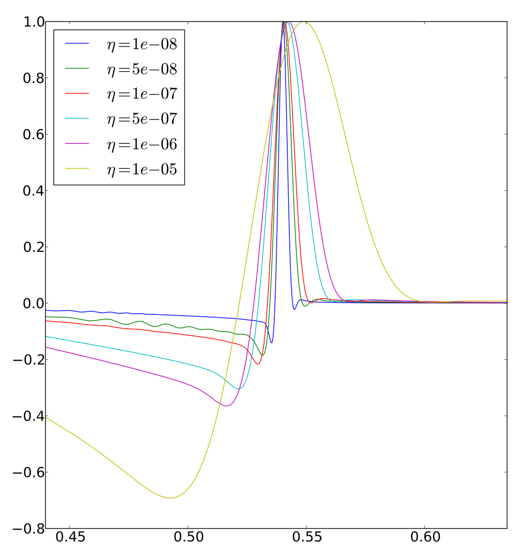


Growth Rates

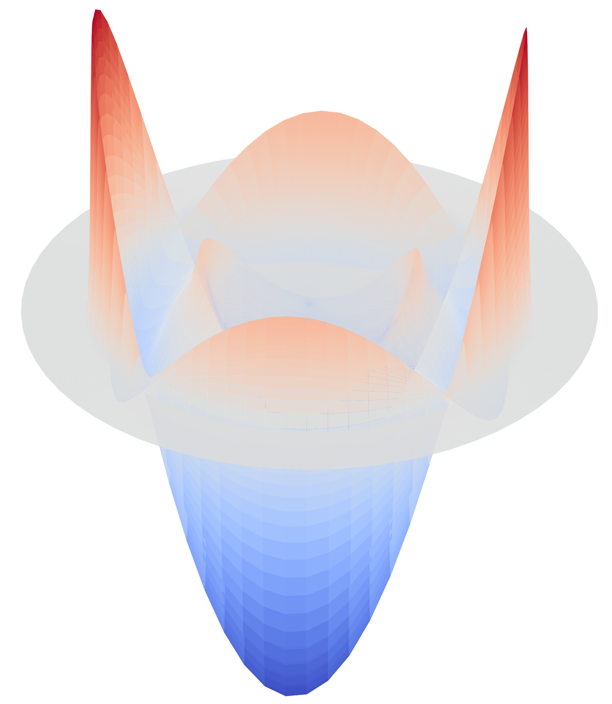


Ideal Current Sheets

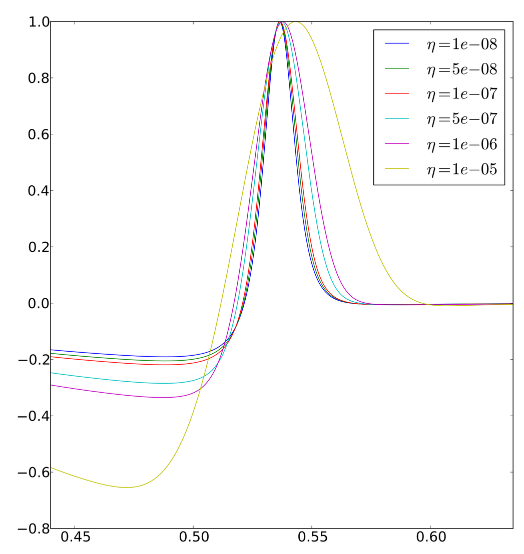
Linear DTM Simulation Results



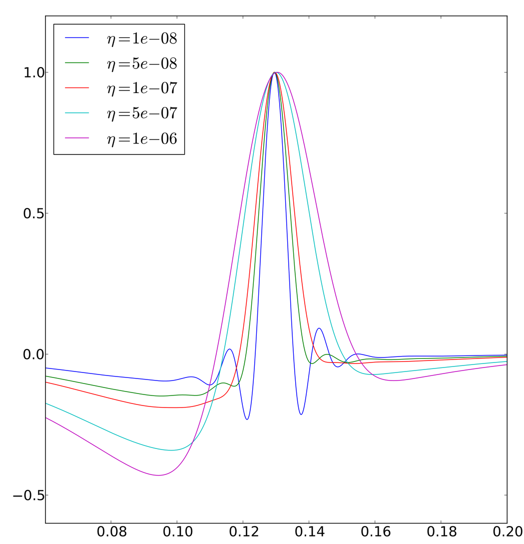
Cyl. Force Free Current Sheets



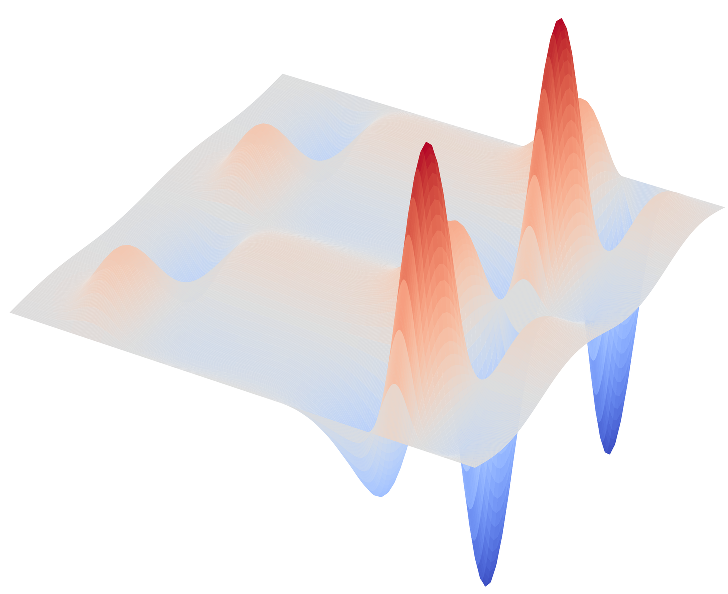
Cylindrical Currents



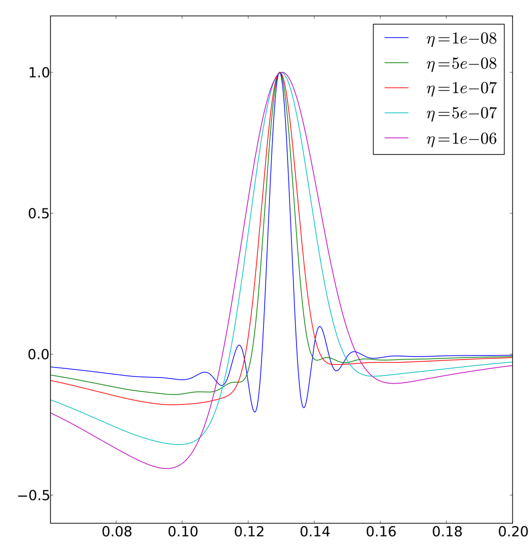
Cyl. $\delta_N = 0.05$ Current Sheets



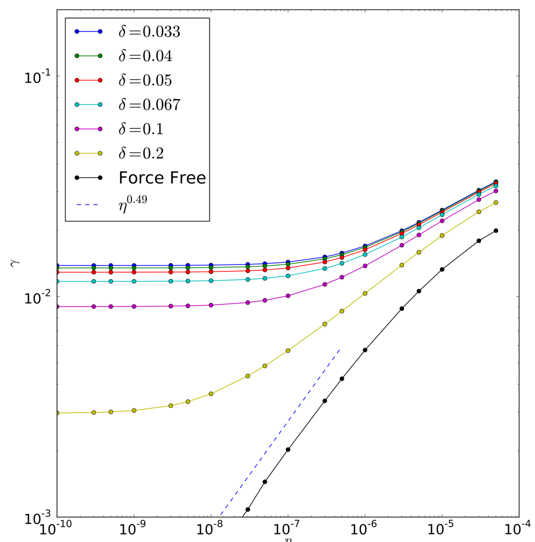
Cart. Force Free Current Sheets



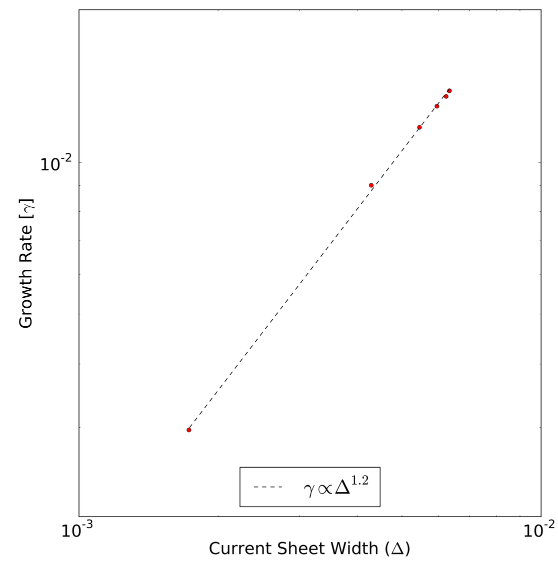
Cartesian Currents



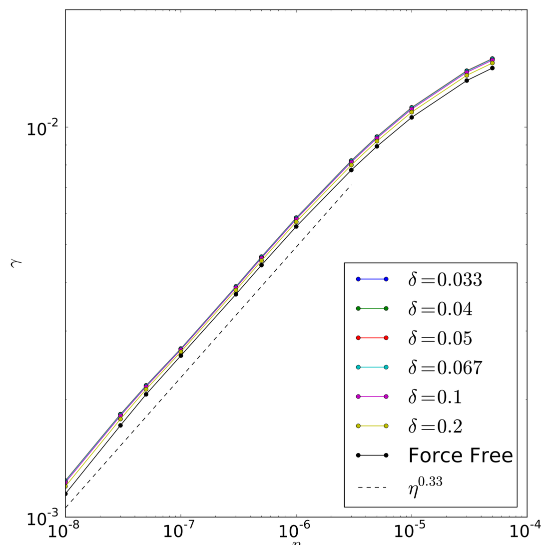
Cart. $\delta_N = 0.05$ Current Sheets



Cylindrical Growth Rate Scaling

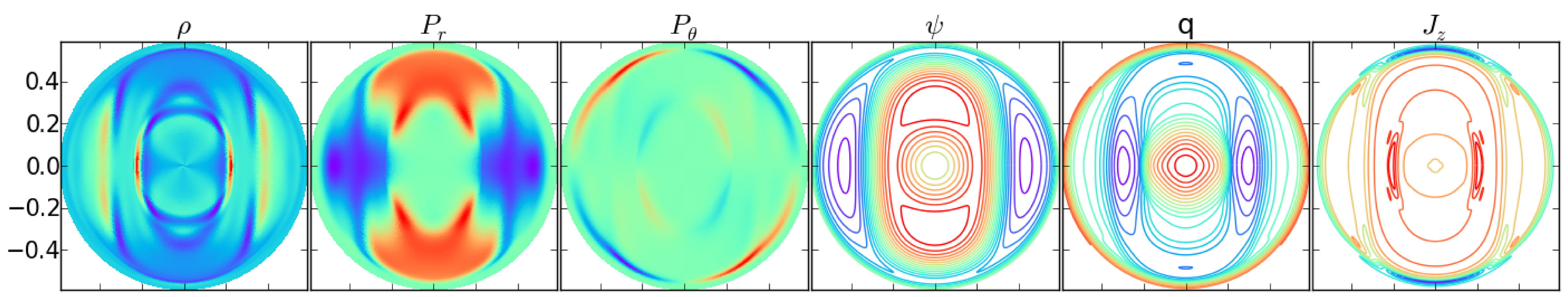


γ Scaling with Saturated Current Sheet



Cartesian Growth Rate Scaling

Nonlinear Results



MRC-3D Model

$$\mathbf{E} = -\mathbf{v}_i \times \mathbf{B} + \frac{d_i}{n}(\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J} - \eta_2 \nabla^2 \mathbf{J}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v} - D \nabla \rho) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + \mathbf{I}(p + B^2/2) - \rho\nu\nabla\mathbf{v}] = 0$$

$$\frac{\partial T_e}{\partial t} + \mathbf{v} \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \mathbf{v} = 0$$

$$p_s = \rho T_s, \quad p = p_e + p_i = (1 + \tau)\rho T_e$$

Where $\tau = T_i/T_e$, $\gamma = 5/3$ for an adiabatic equation of state, η is the magnetic resistivity, $d_i = 0$ is the ion inertial length , ν is the fluid viscosity, and D is a particle diffusivity parameter.
Faraday's Law is used to evolve $\mathbf{B}$.

MRC-V3 Simulation Suite

General features of the Magnetic Reconnection Code include:

- Optimized, parallel numerics with the PETSc library
- Non-uniform spatial meshes
- Generalized curvilinear coordinates
- Adaptive explicit and implicit time-stepping
- Portable `hdf5` data output

Additionally, the specific code used in this work, `mrc-3d`, features:

- Python based bode generation
- Runtime configurable Settings
- Python bindings to internal C routines
- 1D linear and 2D nonlinear implementations

Future Work

The work presented here suggests double tearing modes may couple to a resistivity-independent instability in cylindrical geometries. To better understand this behavior:

- Simulate a wider range of pressure gradient scale heights and lengths (δ_N, N_b), and introduce temperature gradients of the same form.
- Vary the functional form of the pressure profile
- Examine the dependence of the saturated current sheets on the poloidal and axial mode numbers (m, n)

Further high-resolution nonlinear simulations and the introduction of Hall physics will provide insights into:

- Differences between the 'explosive' late nonlinear phase in cartesian and cylindrical geometries
- The potential of diamagnetic effects to stabilize the DTM, similar to the quasi-linear stabilization of the $m = 1$ kink tearing mode [4]
- Viability of comparison to our previous Particle-in-Cell simulations

References

[1] P. L. Pritchett, et al. "Linear analysis of the double-tearing mode" *Phys. Fluids*, **23** , 1368 (1980)

[2] A. Bierwage, et al. "Dynamics of resistive double tearing modes with broad linear spectra" *Phys. of Plasmas* , **14** , 022107 (2007)

[3] X. M. Zhao, et al. "Double tearing modes in the presence of internal trasport barrier" *Phys. of Plasmas* , **18** , 072506 (2011)

[4] B. Rogers & L. Zakharov "Nonlinear ω_*-stabilization of the $m = 1$ mode in tokamaks" *Phys. Plasmas*, **2** , 3420 (1995)