# Spatially Localized Solutions of Shear Flows

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## Goals

- To compute spatially localized equilibrium and traveling wave solutions of some common shear flows.
- To understand the relationships between the new localized solutions and previously computed spatially periodic solutions.
- To use these solutions to gain insight into the structure of turbulent shear flows.

## Introduction

**Coherent structures in turbulence.** One of the primary approaches to understanding the spatiotemporal complexity of turbulence has been to try to break it down into its most fundamental component parts or coherent structures. From an understanding of these parts and how they interact, we hope to be able to understand and reconstruct the properties of turbulent flows. One approach to coherent structures has been to identify them with invariant solutions of the Navier-Stokes equations. The most elemental invariant solutions are equilibria and traveling waves (solutions that are constant in time or translate at a constant rate, respectively).

**Benefit of localization.** Due to the computational intensity of computing these solutions, work has primarily been restricted to small, periodic domains. The computation of solutions localized within a spatially extended domain will have the benefit of allowing these structures to assume their natural dimensions without the artificial constraint of imposed periodicity. This in turn will allow a more informed comparison of these invariant solutions with the coherent structures observed in experiment and numerical simulation.

**Plane Couette and channel flow.** The system we have studied consists of fluid contained between two walls. Motion in the fluid is driven by relative motion of the walls (plane Couette) or by an imposed pressure gradient (channel). These flows are simple representatives of many flows in which fluid is forced past a surface.

**Computation of solutions.** To find invariant solutions we use an iterative search algorithm on a fully-resolved numerical solutions of the flow, which produces an exact soltuion of a nonlinear equation in $10^6$ free variables. For the search to succeed, an initial guess is required that is already close to an invariant solution. We were able to create these guesses from previously computed periodic solutions through a windowing procedure, leading to interesting questions about the relationship between periodic and localized solutions.

## Spanwise localized solutions of plane Couette flow

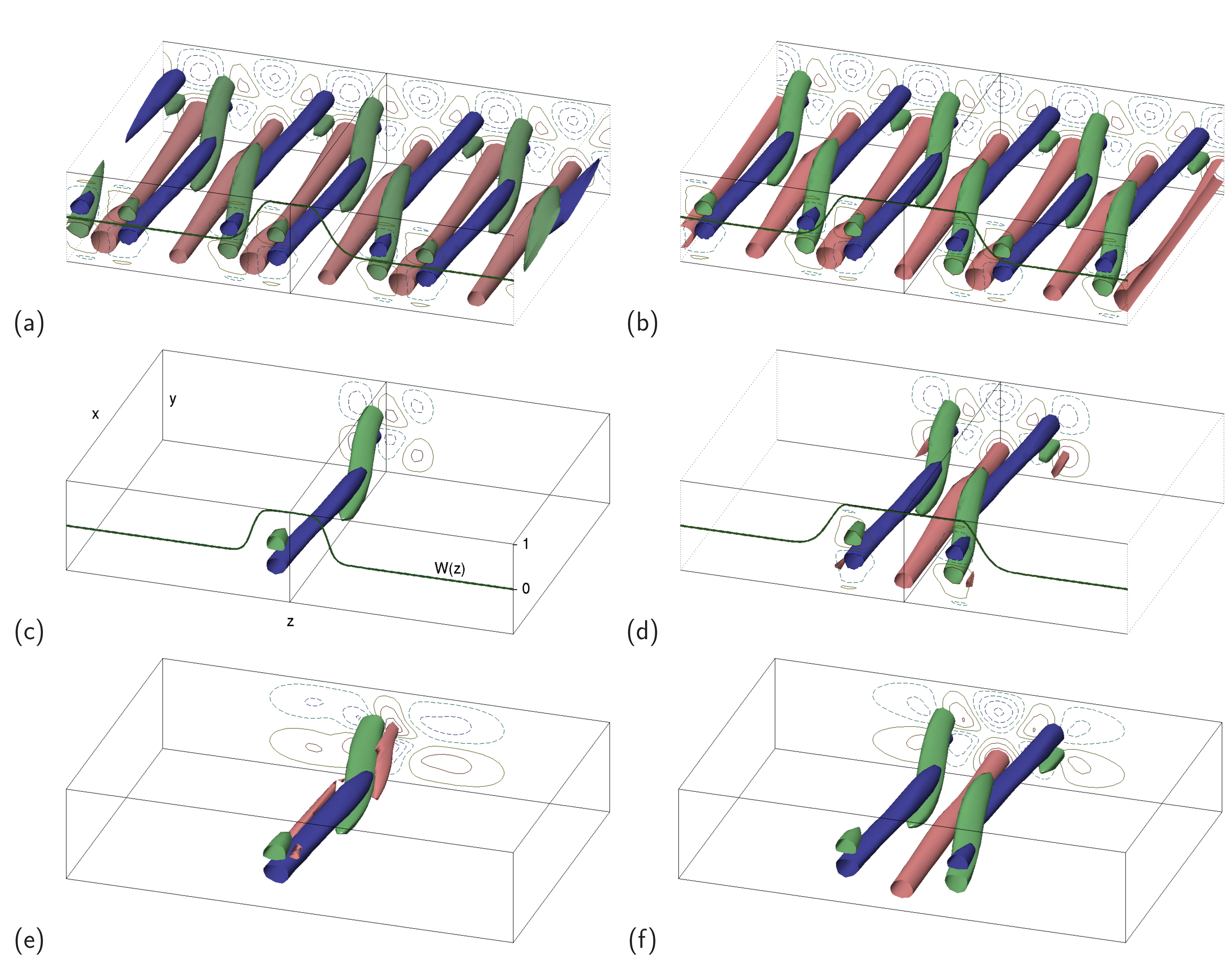


Figure 1: EQ7, a known periodic solution (a,b) is subjected to two different windowings to produce localized initial guesses (c,d). These guesses are fed into the search algorithm which converges on equilibrium solutions (e,f). The blue and green are isosurfaces of signed swirling strength. The red (and yellow in other figures) are isosurfaces of positive (and negative) streamwise velocity. Only the positive isosurfaces are shown here. The contours on the back wall are of streamwise velocity.

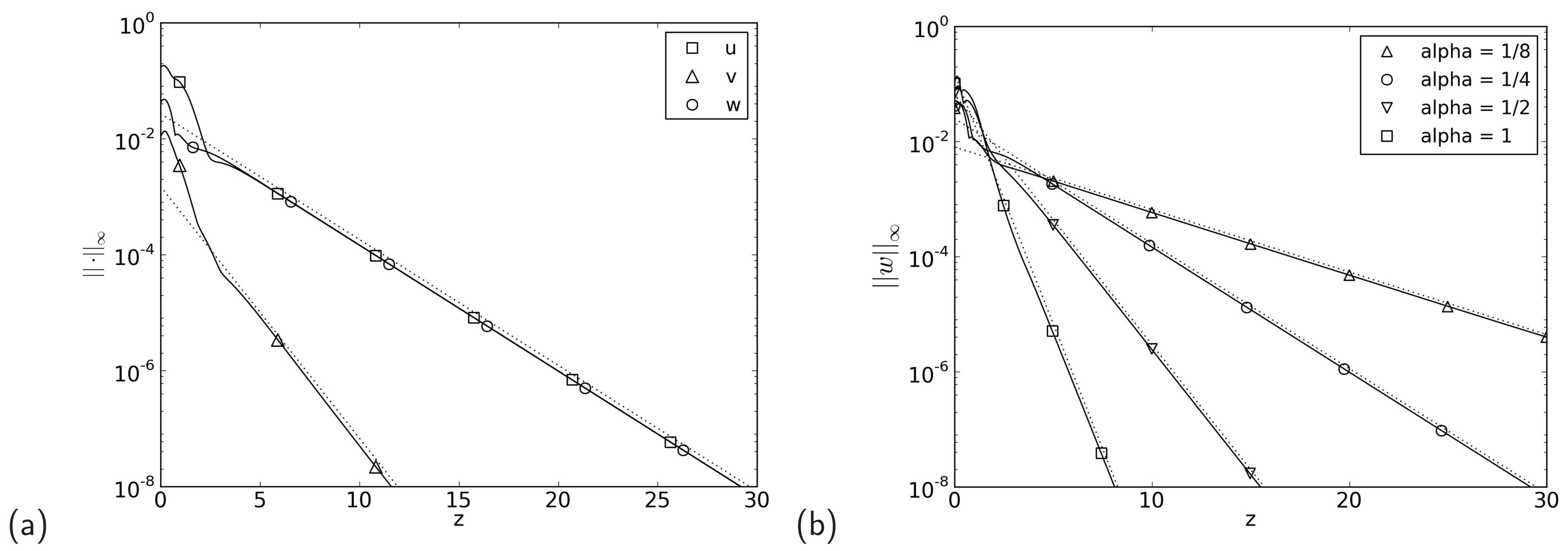


Figure 2: Exponential decay in the tails of spanwise localized plane Couette equilibria.

## Spanwise and wall-normal localized solutions of channel flow

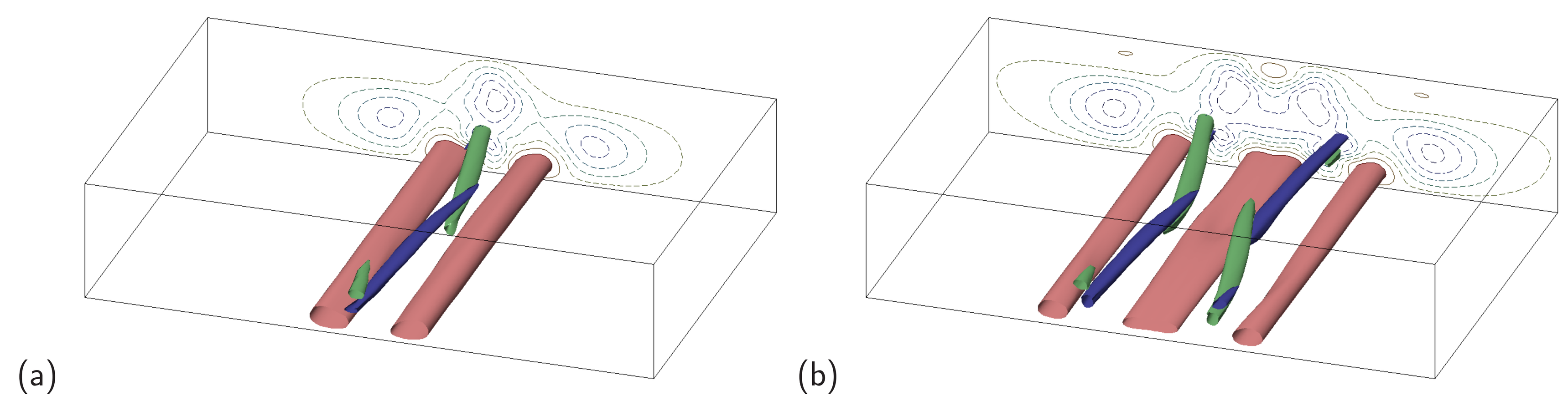


Figure 3: Spanwise and wall-normal localized traveling wave solutions of channel flow. Plotting conventions are as in Figure 1.

## Spanwise and streamwise localized solutions of plane Couette flow

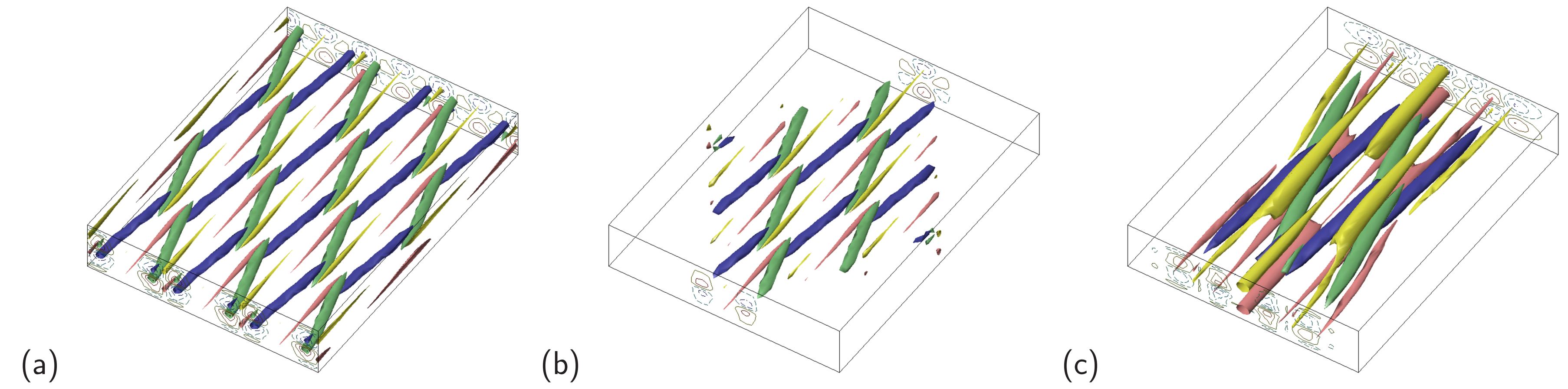


Figure 4: **Initial windowing** starting with (a) 6 copies of the periodic EQ7 arranged in a 2x3 array. This was then windowed to produce (b), the starting guess for the Newton-hookstep search algorithm, which converged on (c) an equilibrium solution.

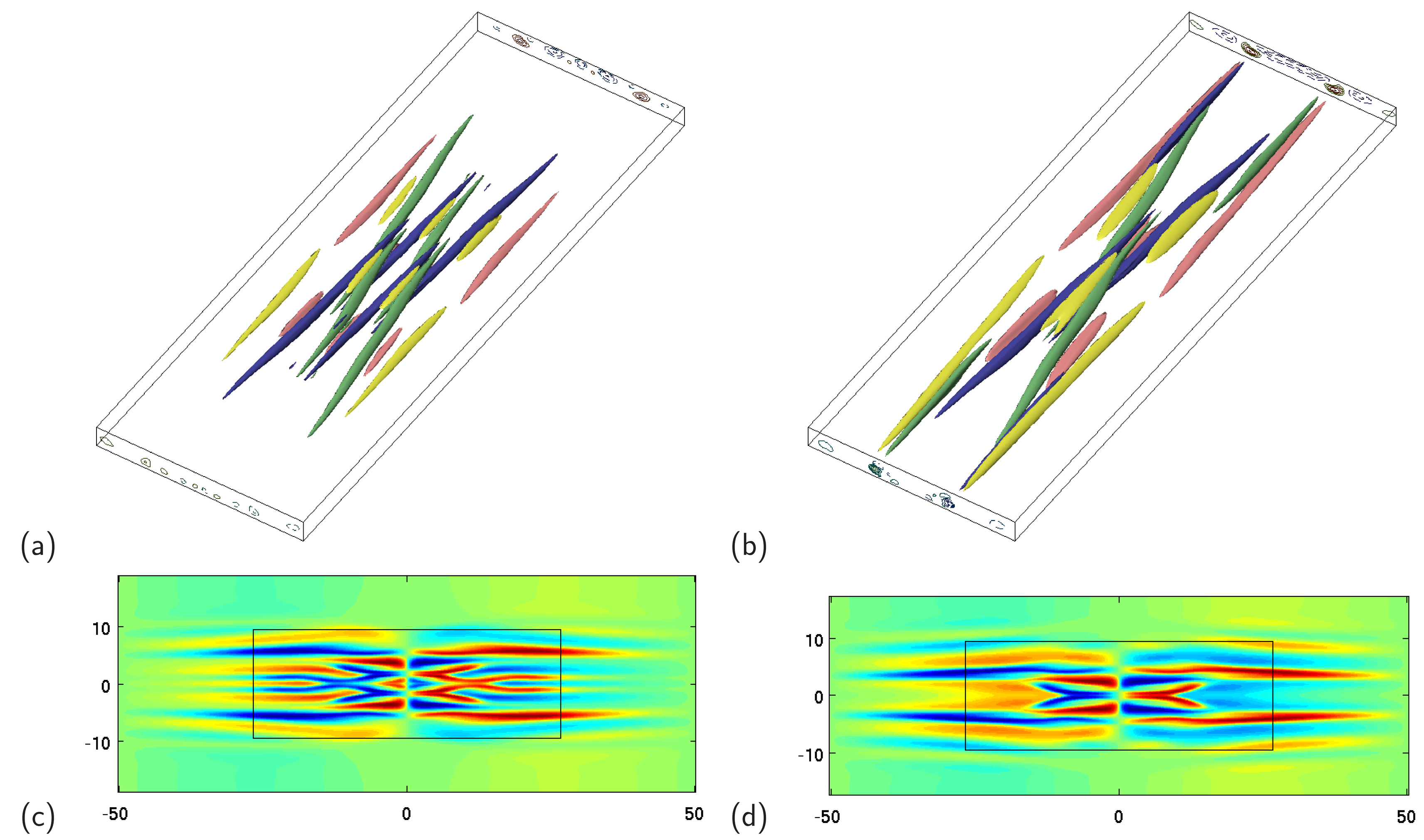


Figure 5: **(a,b) Two doubly local solutions** found after a sequence of windowings similar to that in Figure 4 in increasingly large domains. (c,d) show streamwise velocity in the $y = 0$ plane. The black rectangle denotes the portion of the flow visualized in (a,b). Note the similarity between the arrangement of vorticity in this solution and EQ7, despite the dramatic difference in size.

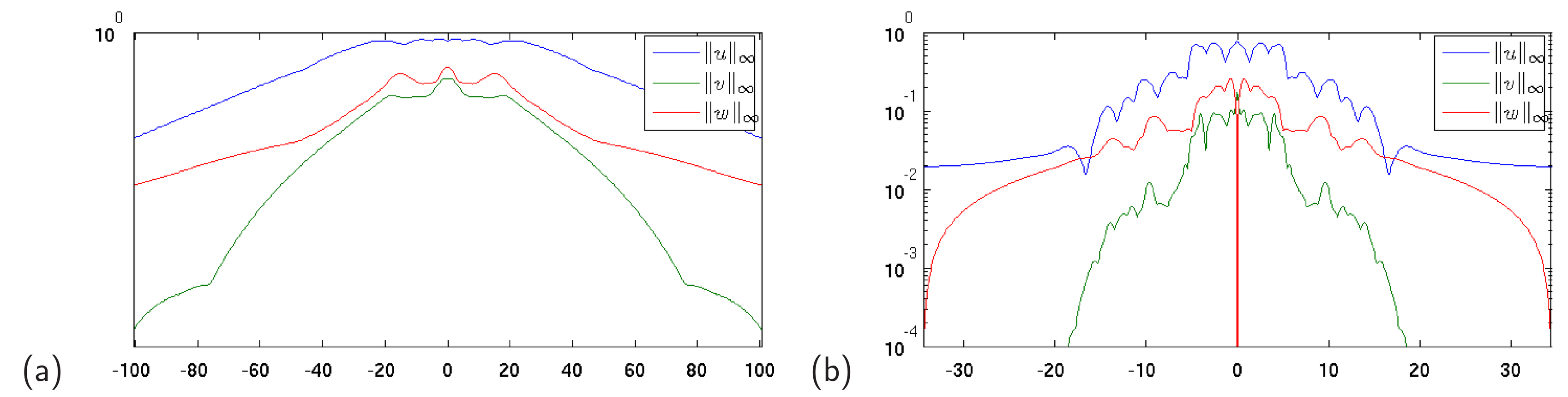


Figure 6: **Tails** showing the decay to laminar of the solution shown in Figure 5(b).

## Conclusions/Future Work

We have made substantial progress in computing localized solutions, although more should be computed to help determine the universal properties. We have begun to address the relationship between periodic and localized solutions through an analysis of the “tails” of localized solutions. There is a large amount of work remaining in understanding the role these solutions play in turbulence. The creation of a dynamic model based on these solutions and their unstable manifolds would be a large step forward.