# Phenomenology of a Toy Model Inspired by a Statistical Curiosity in Quantum Gravity

Patrick Greene
Department of Physics, University of New Hampshire, Durham, NH

## Quantum Gravity:

- The tools of quantum mechanics, which successfully describe three of the four fundamental forces, fail to provide a complete theory when applied naively to gravity.
- Theories of Quantum Gravity attempt to provide a complete quantum theory which incorporates gravity.
- Quantum mechanics often creates discrete measurements, and naturally some models of quantum gravity also make space-time discrete.

## Lorentz Invariance:

- In relativity, the time and space coordinates of moving observers in relation to each other are described by Lorentz Transformations.
- Physics has been experimentally shown to remain unchanged by these transformations. This property is called Lorentz Invariance.

## A Statistical Curiosity:

- Random walks are processes in which random steps are taken such that each new step has no dependence on the previous step.
- When performing a random walk, the probability that the walker returns to his or her starting point is characterized by the spectral dimension.
- In several models of quantum gravity, the spectral dimension has been observed to change at small scales.

## Inspiration for This Toy Model:

- In a number of quantum gravity models, the spectral dimension changes from 4 at large scales to 2 at small scales.
- We thus study particle propagation on a space-time made from 2D surfaces (triangles) embedded in 4D space in a Lorentz Invariant manner.

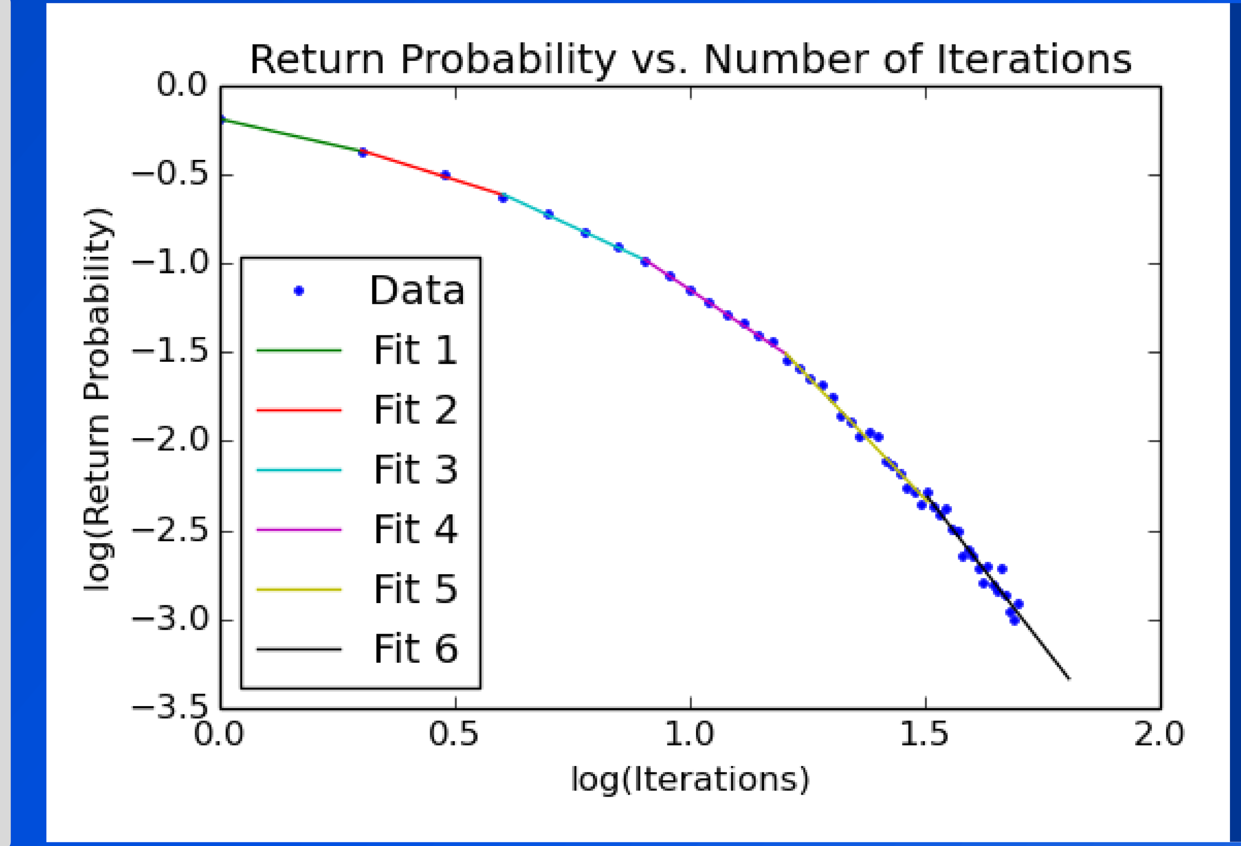


**Figure 1:** The dots correspond to probabilities of a particle returning to the origin. The lines correspond to fits of the slope on the log-log plot, which in turn is proportional to the spectral dimension.

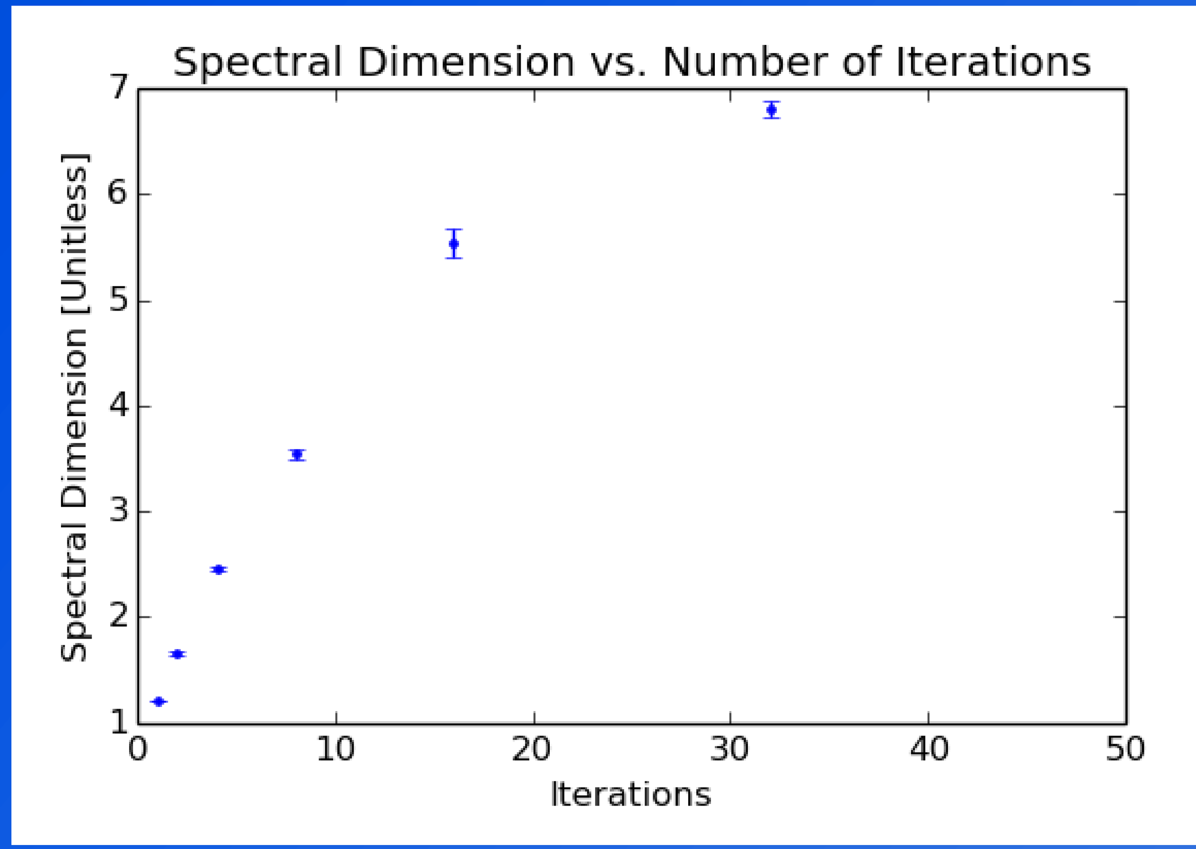


**Figure 2:** The spectral dimension grows past 4 after just 20 iterations. This shows that this type of Lorentz Invariant 2D discretization cannot reproduce our macroscopic spectral dimension.

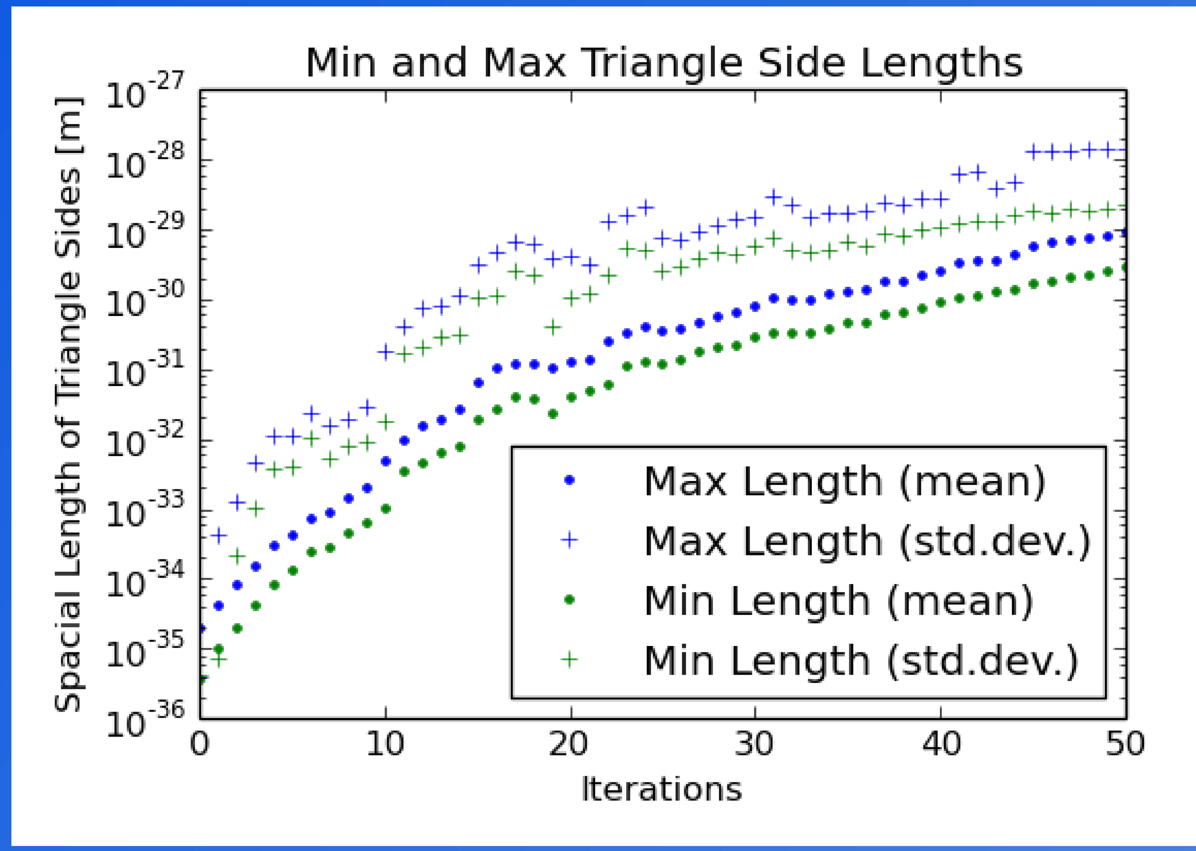


**Figure 3:** The minimum and maximum lengths of the spacial projections of the sides of the triangles grow exponentially. Note that the standard deviations are much larger than the means.

## References:

S. Carlip, *Spontaneous Dimension Reduction*?, July 2012. **arXiv:1207.4503**

A. Eichhorn and S. Mizera, *Spectral Dimension in Causal Set Quantum Gravity*, Nov. 2013. **arXiv:1311.2530**

## Key Questions:

- Will we have reasonable behavior in the at large scales?
- Can we preserve Lorentz Invariance while also avoiding nearly null triangles?

## My work:

- A particle traveling on a triangle will eventually see an edge and move on to a new triangle.
- We assume that a particle moves unhindered in a straight line along a given triangle, so any random walk characteristics are due to the random nature of the space-time.
- We generate new triangles off of previous triangles in a Lorentz Invariant manner, which requires that the next triangle seen by the particle has certain properties.
- We repeat this process and from the results calculate the return probability.
- Figure 1 shows a plot of the return probability as a function of iterations. The spectral dimension, given by Equation 1 below, can be approximated at different scales from the slopes in the log-log plot.

$$d_s = -2\frac{\partial \ln(P_n)}{\partial \ln(n)}$$

**Equation 1:** This is the relation between the spectral dimension and the return probability. It means that the spectral dimension is related to the slope of the log-log plot in Figure 1.

## Results:

- As shown in Figure 2, our spectral dimension grows to well above 4 after about 20 iterations.
- Furthermore, as shown in Figure 3, at any particular point, the triangles become much larger further from the origin, due to the Lorentz Invariance we imposed.
- We thus conclude that Lorentz Invariance can be preserved in this toy model of quantum gravity only by losing translational invariance and an incorrect macroscopic spectral dimension.