The Toric Approach to F-theory Model Building

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Background

String theory is the leading candidate for a fundamental theory of physics that may provide a unified microscopic description of nature. One of the most important features of string theory is the requirement of a ten dimensional spacetime. F-theory generalizes the string theory framework by incorporating the complex string coupling field as two additional dimensions. This results in an eight dimensional space representing six physical dimensions and two associated to the string coupling, which is assumed to be small and compact. The geometries that describe the possible shape of the hidden dimensions are called elliptically fibered Calabi-Yau fourfolds, and their geometries significantly impact the properties and interactions of the particles we observe.

Research Questions

What are the geometric properties of elliptically fibered Calabi-Yau manifolds?

How do these properties affect the physics that we observe?

Methods

Construct elliptically fibered Calabi-Yau manifolds using available data on reflexive polyhedra [1].

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Generalize existing analytical tools to calculate important properties of the Calabi-Yau manifolds.

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Use known relationships between geometry and physics to understand the physical implications of each model.

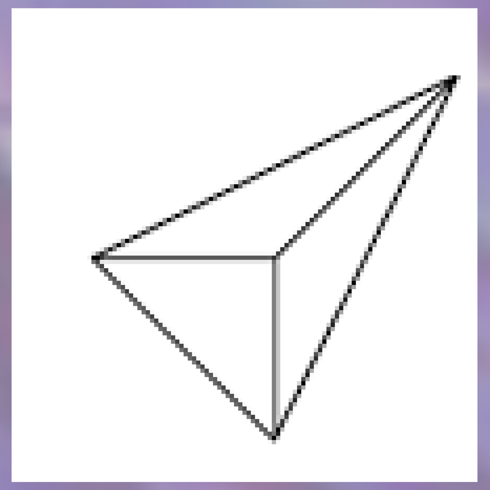
Motivation

Many of the ideas needed to analyze elliptically fibered Calabi-Yau manifolds are well established, but there remains a wealth of knowledge to be uncovered about them. Our approach is to systematize the analysis of these manifolds, allowing observations to be made involving entire classes of manifolds and their resulting physics. We have made extensive use of simple models as a laboratory to test our ideas. Ongoing related research by the authors and a group at MIT suggests the connection between geometry and physics is even deeper than previously thought [2].

Constructing elliptically fibered Calabi-Yau manifolds can be done using a database of polytopes, encoding geometric data (see below), analytical tools developed by the authors for the study of Calabi-Yau manifolds adaptable to the analysis of elliptically fibered Calabi-Yau manifolds, and mathematical tools that relate the manifolds to the four-dimensional physics which have been developed over decades of research on Calabi-Yau manifolds in general.

Construction of Calabi-Yau Manifolds

In toric geometry, reflexive polytopes defined on an integer lattice are used to represent geometric data for Calabi-Yau manifolds. These are closed and convex polygons which contain only the origin as an interior point. Roughly speaking, relationships between the points are used to construct a polynomial equation that describe the Calabi-Yau manifold [3]. In F-theory, the two dimensions that describe the string coupling are allowed to change shape at different points on the six physical dimensions. This is described by the Weierstrass equation:



One of 16 two-dimensional reflexive polytopes.

$$y^2 = x^3 + x * f(z_i) + g(z_i)$$

Here, x and y are (complex) coordinates describing an elliptic curve, associated with the string coupling, which along with the z_i of the remaining six dimensional space, is the elliptically fibered Calabi-Yau manifold.

Present Work – Geometric Analysis

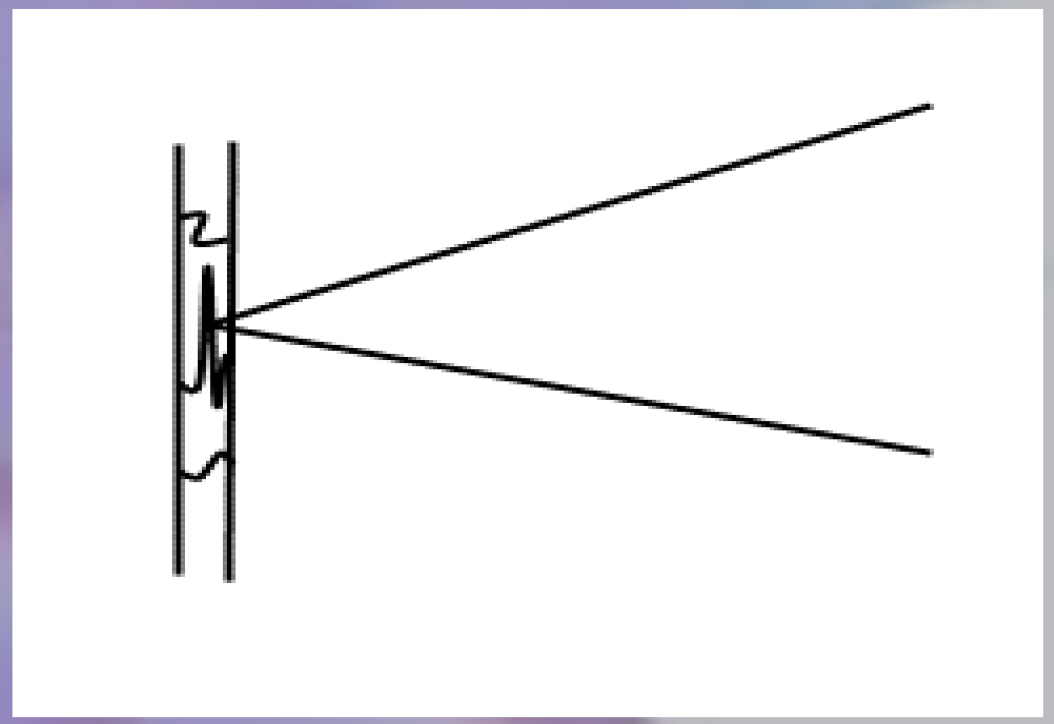
Existing analytical tools developed by the authors are being adapted to analyze the geometry of Calabi-Yau manifolds obtained by the above procedure. Important properties include Hodge numbers (roughly the number of holes in the manifold) and the volume of the holes, as well as the Calabi-Yau manifold itself [4]. These quantities are needed to understand the physics that we would observe for a given configuration of the hidden dimensions.

Expected Results – Physics, Tools

We expect two important results from this project:

1. An understanding of simple models of the hidden dimensions in the context of F-theory.
2. A set of tools for understanding more realistic, and hence complicated, models.

We expect the toy models currently under study to exhibit simplified, non-realistic physics. In particular, they lack singularities where the two dimensions of the elliptic fiber, representing the string coupling, "pinch off" to a point, leading to charged particle content and gauge interactions. Mathematically, this occurs when f and g in the Weierstrass equation take a specific form. Physically, this is due to physical objects called D-branes, on which strings end, stacked at a point. The strings bound to these D-branes give rise to generalizations of observed particle interactions.



Particle interactions due to D-branes stacked at a point. D-branes are the physical cause of singularities of the elliptic fiber.

Future Work

This project will lead naturally to the study of more complicated elliptically fibered Calabi-Yau manifolds which may resemble the physics of our universe. A more detailed analysis of the toy models studied here, as well as more complicated models, is expected to shed light on relationships between geometry and physics that are only beginning to be understood. Work with MIT researchers has already been initiated to explore this avenue of research [2].

References

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[3] A. Collinucci, R. Savelli. *On Flux Quantization in F-theory.* JHEP 1202 (2012) 015.
[4] P. Berglund, P. Mayr. *Heterotic String / F theory Duality from Mirror Symmetry*. Adv. Theor. Math. Phys. 2 (1999) 1307-1372.

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