Rates of Magnetic Reconnection at a Separator

with Deference to Coronal Heating

Ian William Hoppock

University of New Hampshire & University of St Andrews

Abstract

Through a model that relies on a magnetic ring formed via Faraday's Law and helicity along a localised separator (i.e., the nulls are considered far enough away), a magnetic reconnection regime is analytically created. The purpose of which is to take a natural phenomenon of the corona and relate it to the coronal heating problem. The model shows that separators, like nulls, bifurcate when the angle between fan planes decreases enough. Then, upon following the evolution of magnetic flux, entirely distinct magnetic domains emerge. However, the rate of reconnection experiences diminishing returns. Finally, modifying the model (e.g., breaking the symmetry) shows a modification of the reconnection rate.

Introduction

A separator is the field line that connects two nulls. Briefly, a null consists of a spine and a fan plane. The generic (most stable) separator is the intersection of two fan planes. Thus, a positive and negative null are needed (otherwise, a spine would have to lie indefinitely in a fan plane, which is not possible given the perturbations in the solar atmosphere; moreover, separators have been shown to be stable.

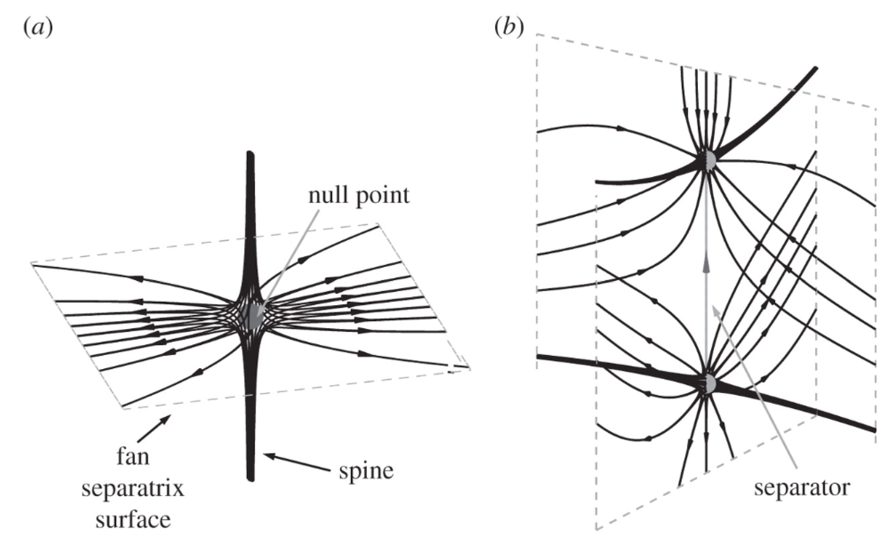

Figure 1: Neutral point and generic separator.

Reconnection is possible at the separator (away from the nulls) if there is a strong enough current (rights side) such that, from a cross sectional view, there exists an elliptical field. Separators have been shown to be especially common in the solar corona; we attempt to illustrate one plausible scenario contributing to coronal heating. However, for this to be a sufficient contribution, we must determine the frequency with which this process occurs; therefore, the reconnection rate is essential.

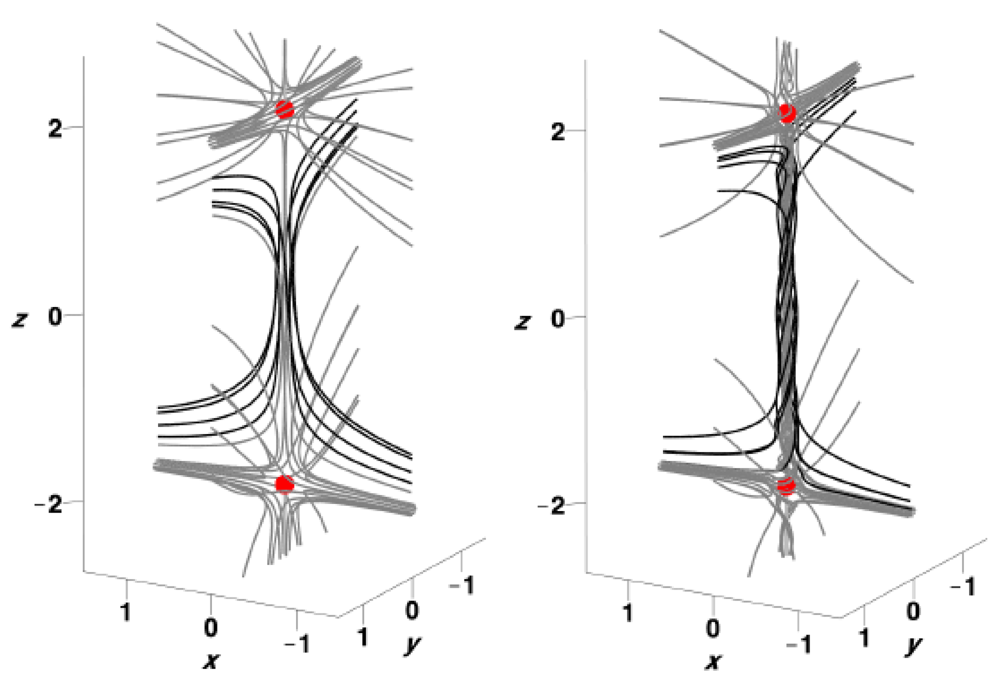

Figure 2: On the left, a cross section will yield a hyperbolic field where separator reconnection will be unlikely. On the right, we see that, due to an amplified current, a twisting of magnetic field lines around the separator (allowing for a cross section to be elliptic) which can provide for a reconnection regime. *Nota Bene: the fan and spine are visible here.*

The Model

Generally, we have a magnetic field in which there exists a separator. Then a ring shaped magnetic field is positioned around the separator. This facilitates reconnection:

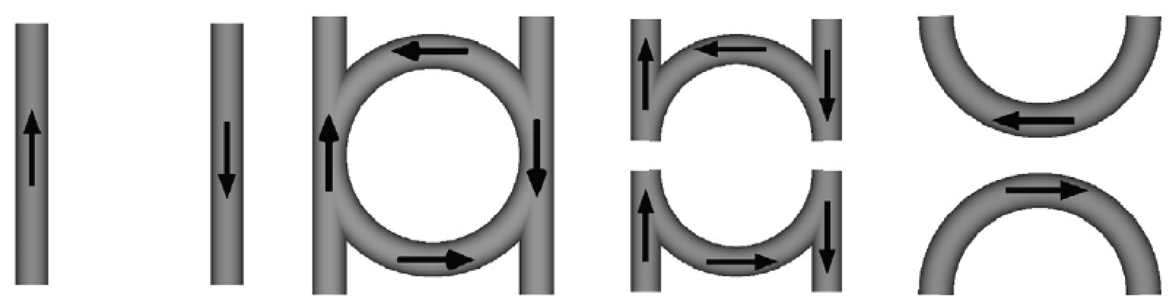

Figure 3: Two antiparallel field lines upon which a magnetic ring is placed: reconnection proceeds.

The evolution of these magnetic fields yields an electric field upon which we can consider the changes in the magnetic topology. Our equations are:

$$\mathbf{B}_0 = \frac{b_0}{L^2}\left(x(z-3z_n)\hat{\mathbf{x}} + y(z+3z_n)\hat{\mathbf{y}} + (z_n^2 - z^2 + x^2 + y^2)\hat{\mathbf{z}}\right)$$

$$\mathbf{B}_r = \nabla \times \left(b_r a \exp\left[-\frac{(x-x_c)^2}{a^2} - \frac{(y-y_c)^2}{a^2} - \frac{(z-z_c)^2}{l^2}\right]\hat{\mathbf{z}}\right)$$

$$\mathbf{E} = b_r a \exp\left[-\frac{(x-x_c)^2}{a^2} - \frac{(y-y_c)^2}{a^2} - \frac{(z-z_c)^2}{l^2}\right]\hat{\mathbf{z}}$$

This is due to our field evolution: from time 0 to 1, we have

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_r t$$

Separators Birurcate

When this process becomes exceptionally strong, i.e., the magnetic field is severely elliptic, just like nulls, separators will be created in pairs, creating distinct magnetic domains:

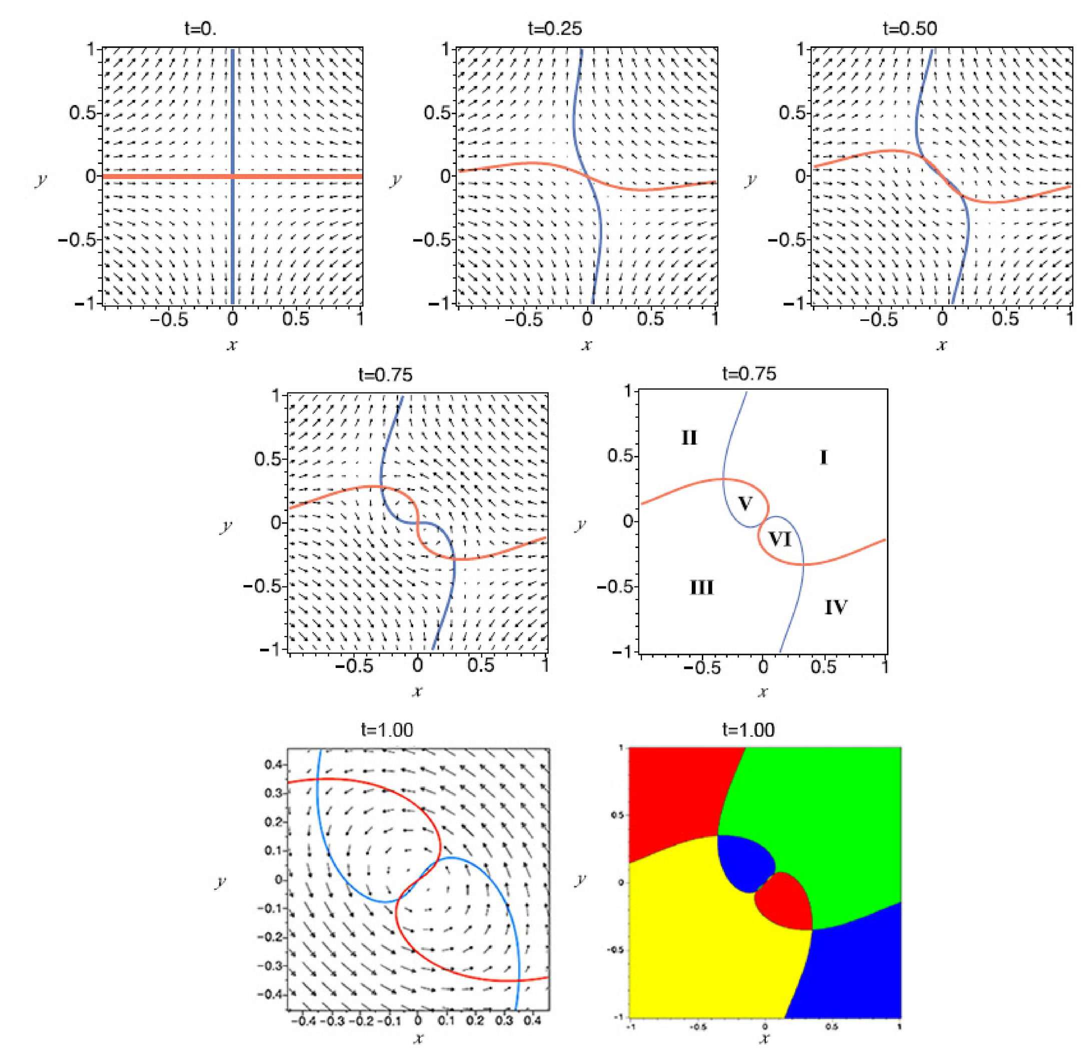

Figure 4: Several cross sections (xy-plane) of our field at certain times t. The lines are the fan planes. Over time, they intersect forming new separators (as a separator is the intersection of two fans). After some time, we have six distinct magnetic domains. Then, very close to the end of the simulation, two more separators emerge. The colouring describes the relationship between the outer magnetic domains and the ones ensconced by the separators. I.e., the magnetic field lines came from that part of the field. Tracing individual magnetic field lines gives this result, though it is not done here.

Evolution of Flux

In ideal MHD the field lines are frozen into the plasma and magnetic topology is conserved. However, in the non-ideal system (i.e., inside the ring) we take the current to be a potential in Ohm's law:

$$\mathbf{E} + \mathbf{w} \times \mathbf{B} = \mathbf{R} = \nabla\Phi$$

Because the electric and magnetic fields are not perpendicular, the field lines are not frozen in. Thus, we must work from the boundaries of our system, where the electric field vanishes, via a transverse surface, $\mathbf{r}$:

$$\Phi(\mathbf{r}(k)) = \int_{k=0}^{k} E_{\parallel}(\mathbf{r}(k))\mathrm{d}k$$

where

$$\frac{\mathrm{d}\mathbf{r}(k)}{\mathrm{d}k} = \frac{\mathbf{B}}{B}$$

Thus,

$$\mathbf{w} = \frac{\mathbf{B} \times (\nabla\Phi - \mathbf{E})}{B^2}$$

and upon complete integration, we have an exact value:

$$\mathbf{w}^* = \frac{\mathbf{B} \times \nabla\Phi}{B^2}$$

Moreover, following independent field lines illuminates the ways the topology evolves.

Rate of Reconnection

The following is the rate of change of magnetic flux between the domains:

$$\frac{\mathrm{d}\Phi_r}{\mathrm{d}t} = \left|\int E_{\parallel}(0,0,z)\mathrm{d}l\right|_{max}$$

In this system, we obtain an answer in terms of the error function. However, after bifurcation, some flux is transferred back to its original domain, diminishing the rate of reconnection:

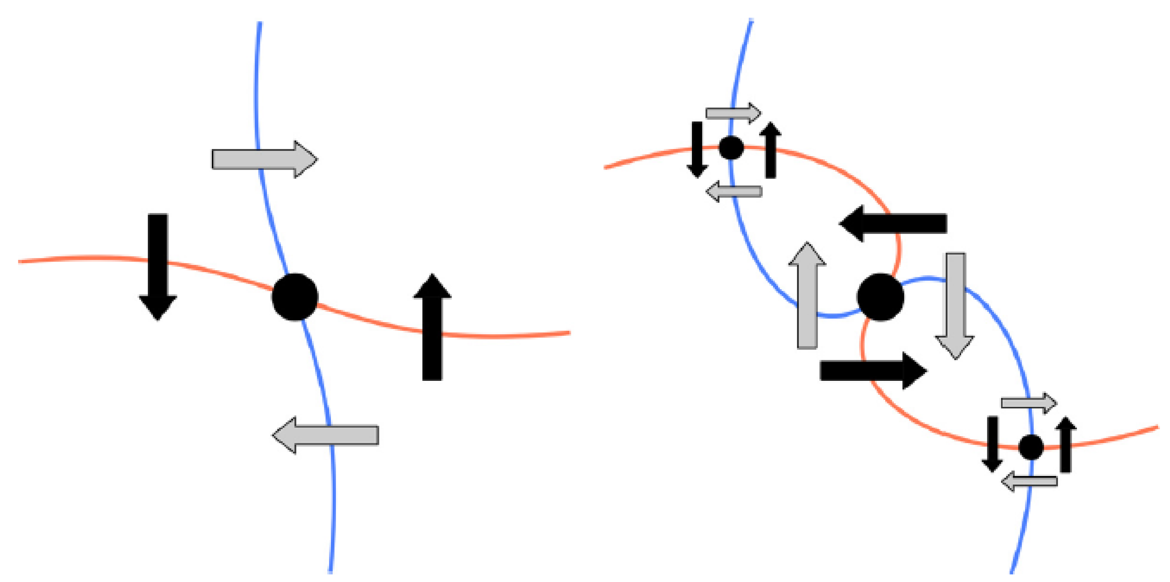

Figure 5: The transport of flux across fans. It is nice to mentally superimpose these images to those on the left.

Modifying the Model

Here, we extend the work of Wilmot-Smith and Hornig by changing the variables of the model. Firstly, we consider changing the height dimension of the ring to make it more cylindrical. This, coupled with variations of the neutral points, modifies the reconnection rate:

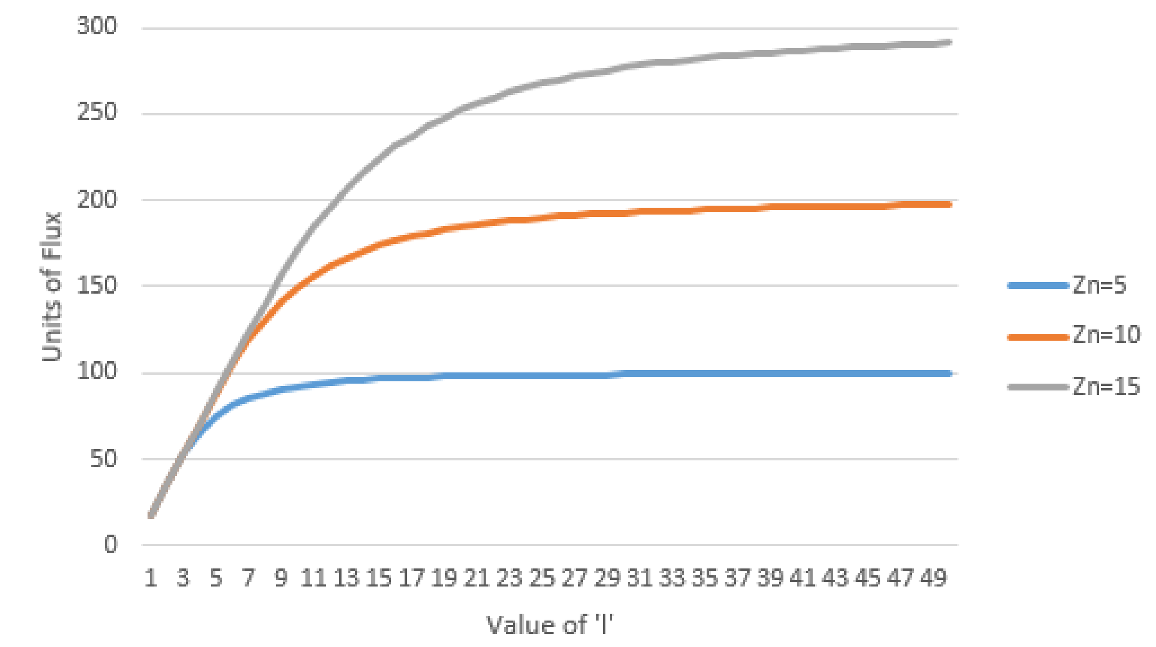

Figure 6: As one can see, just after the neutral point has been exceed by the ring, the rate of reconnection becomes level.

Hence, we see the rate of reconnection increase as the ring becomes more cylindrical, but does approach the limit:

$$\lim_{t\to\infty} \left.\frac{\mathrm{d}\Phi_r}{\mathrm{d}t}\right|_l = b_r z_n$$

As mentioned earlier, the neutral points play no role other than allowing the separator to exist. We can see this here:

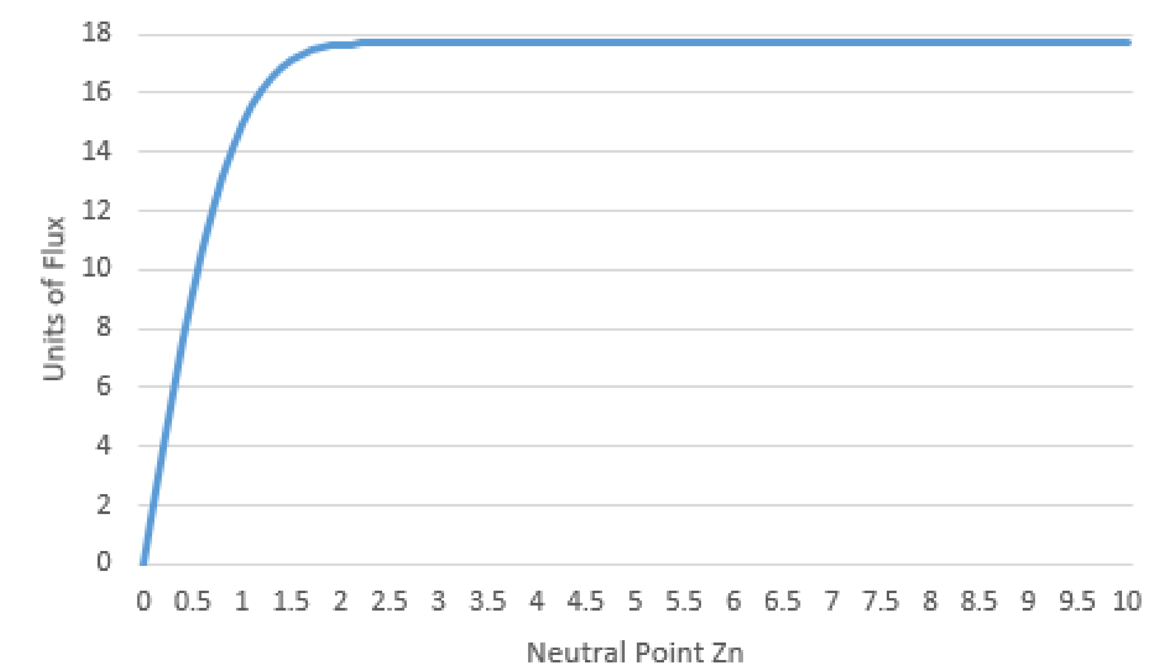

Figure 7: Clearly, separator reconnection vanishes entirely when there is no separator; however, it rapidly increases and then levels. This is to do with the exponential term in the electric field. Here, at $z_n = 2.5$ the electric field is but 0.2% of its maximum at the origin.

Conclusions

Separators have been shown by Parnell *et al.* to be common in the corona. Moreover, since they are also incredible stable elements of the magnetic field, they are part of the magnetic skeleton. Hence, reconnection regimes involving them, which also occur frequently enough, are an excellent source of energy transfer in the corona. Thus, this model is likely to play a role in the overall mosaic of the coronal heating problem. Here, we have shown that the frequency is plausibly high enough to contribute substantial reconnection regimes.

References

[1] A. L. Wilmot-Smith and G. Hornig. A time dependent model for magnetic reconnection in the presence of a separator. *The Astrophysical Journal*, 740:89–101, 2011.