

# Radio Emissions from Streamer Ensembles

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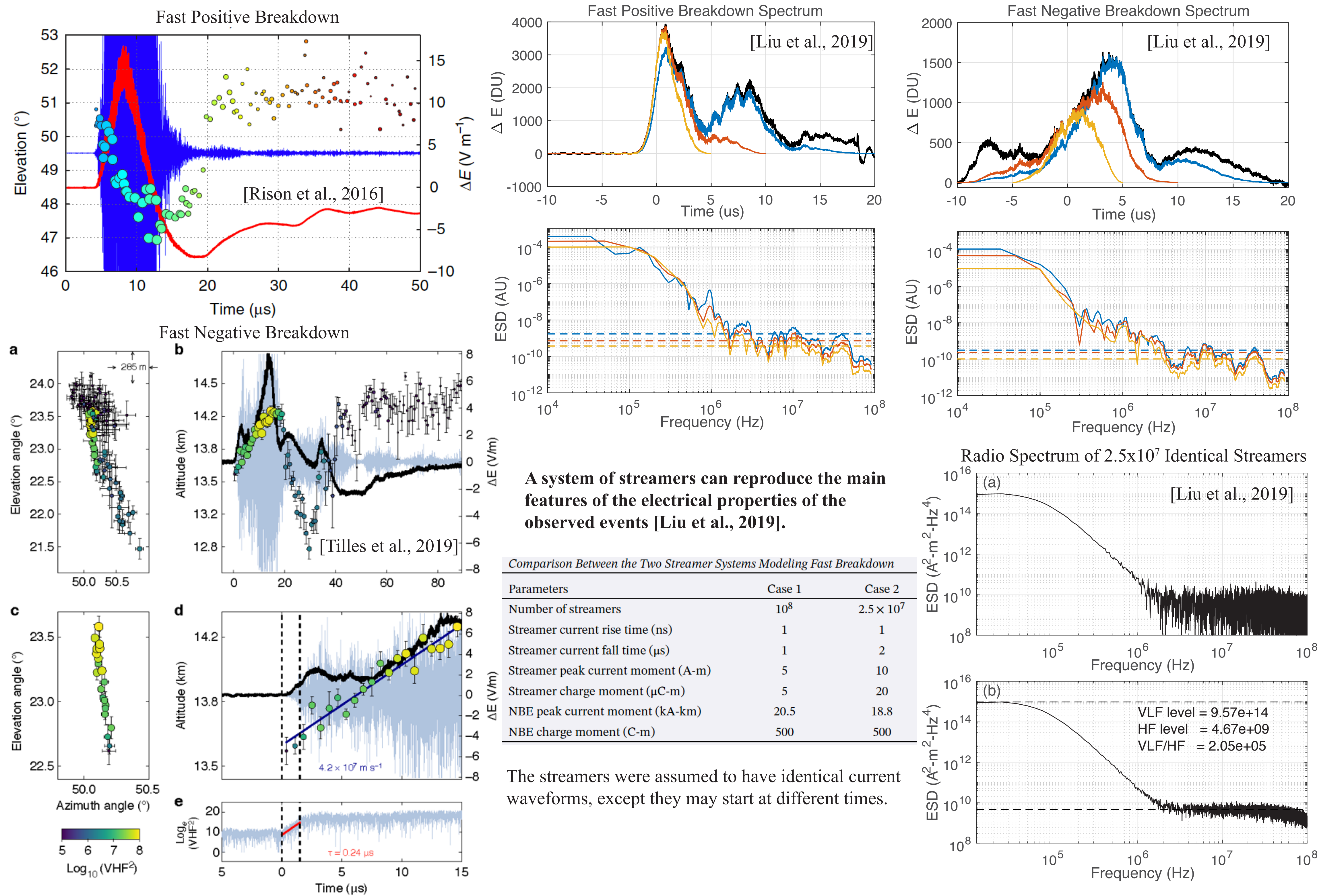
## Abstract

Streamers play important roles in many natural discharge phenomena such as lightning, jets, and sprites. At thunderstorm altitudes, they can generate rapid variations of electric current and are important sources of high frequency (HF) and very high frequency (VHF) radio emissions. Understanding the radio emissions from streamers is therefore critical to interpreting broadband radio observations of lightning. In particular, recent broadband radio interferometer observations have found that narrow bipolar events, powerful sources of HF and VHF radiation, are caused by newly-identified fast breakdown (Rison et al., Nat. Commun., 7, 10721, 2016; Tilles et al., Nat. Commun., 10, 1648, 2019). It has been hypothesized that fast breakdown is a system of streamers (Rison et al., 2016).

In a previous study, a statistical approach was developed to model the radio emissions of ensembles of identical streamers (Liu et al., J. Geophys. Res. Atmos., 124, 10.1029/2019JD030439). Streamer systems in nature, however, are composed of nonidentical streamers, as indicated by their different propagation directions and lengths. In this talk, we discuss an improved approach that can model a system of nonidentical streamers. The improved model can account for random variations among the streamers. For an ensemble of nonidentical streamers, it is still true that the low frequency part of the radio spectrum is determined by the coherent summation of the radio emissions of the individual streamers and the high frequency part by the incoherent summation. The random variation among the streamers can however result in the enhancement of the spectral magnitude in the frequency range that corresponds to the timescale of the variation.

## Introduction

Recent broadband radio interferometer observations have identified that narrow bipolar events (NBEs) are caused by fast breakdown, which is believed to be a system of streamers [e.g., Rison et al., 2016; Tilles et al., 2019; Liu et al., 2019]. Both polarities of fast breakdown can lead to NBEs, and they have similar radio spectra.



## Theory of Radio Spectrum of Nonidentical Streamers System

• Suppose the current moment of a component discharge as a function of  $t$  is  $M_c(t) = M_c(t; \alpha, \beta, \dots)$ , where  $\alpha, \beta, \dots$ , are parameters that represent, for example, rise time, fall time, etc.

• The current moment of an ensemble of  $N$  component discharges is then

$$M_{\text{ens}}(t) = \sum_{i=1}^N M_c(t-t_i; \alpha_i, \beta_i, \dots) = \sum_k \sum_l \sum_m \dots M_c(t-t_k; \alpha_l, \beta_m, \dots) n_{k,l,m,\dots},$$

where  $n_{k,l,m,\dots}$  is the number of the component discharges starting at  $t_k$  with  $\alpha = \alpha_l, \beta = \beta_m$ , etc.

• Let  $\tilde{M}_{\text{ens}} = \langle \tilde{M}_{\text{ens}} \rangle + d\tilde{M}_{\text{ens}}$ , where  $\tilde{M}_{\text{ens}}$  is the Fourier transform of  $M_{\text{ens}}(t)$ ,  $\langle \tilde{M}_{\text{ens}} \rangle$  is its average over many ensembles, and  $d\tilde{M}_{\text{ens}}$  is the deviation of the Fourier transform of a specific ensemble from the average. Assuming  $\langle \tilde{M}_{\text{ens}} d\tilde{M}_{\text{ens}}^* \rangle = \langle \tilde{M}_{\text{ens}}^* d\tilde{M}_{\text{ens}} \rangle = 0$ , i.e., they are uncorrelated,

$$\langle |\tilde{M}_{\text{ens}}|^2 \rangle = |\langle \tilde{M}_{\text{ens}} \rangle|^2 + \langle |d\tilde{M}_{\text{ens}}|^2 \rangle.$$

• Suppose the starting time of a component discharge is described by some normalized distribution  $f_{\text{ens}}(t)$  with  $\int f_{\text{ens}}(t) dt = 1$ , and the set of the parameters for a component discharge follows a normalized distribution  $f_c(\alpha, \beta, \dots)$  with  $\int f_c(\alpha, \beta, \dots) d\alpha d\beta \dots = 1$ .

$$\begin{aligned} \langle \tilde{M}_{\text{ens}} \rangle &= \sum_k \sum_l \dots \tilde{M}_c(\omega, \alpha_l, \dots) e^{-j\omega t_k} \langle n_{k,l,\dots} \rangle \\ &= N \int f_{\text{ens}}(t) e^{-j\omega t} dt \int \dots \tilde{M}_c(\omega, \alpha, \dots) f_c(\alpha, \beta, \dots) d\alpha d\beta \\ &= N \tilde{f}_{\text{ens}} \tilde{M}_c. \end{aligned}$$

• So

$$\langle |\tilde{M}_{\text{ens}}|^2 \rangle = N^2 |\tilde{f}_{\text{ens}}|^2 |\langle \tilde{M}_c \rangle|^2$$

• On the other hand,

$$\langle |d\tilde{M}_{\text{ens}}|^2 \rangle = \sum_k \sum_l \dots \sum_{k'} \sum_{l'} \dots \tilde{M}_c(t-t_k, \alpha_l, \dots) \tilde{M}_c^*(t-t_{k'}, \alpha_{l'}, \dots) e^{-j\omega(t_k-t_{k'})} \langle dn_{k,l,\dots} dn_{k',l',\dots} \rangle.$$

• Assuming  $\langle dn_{k,l,\dots} dn_{k',l',\dots} \rangle \neq 0$  only if  $k = k', l = l'$ , etc.,

$$\begin{aligned} \langle |d\tilde{M}_{\text{ens}}|^2 \rangle &= \sum_k \sum_l \dots |\tilde{M}_c|^2 \langle dn_{k,l,\dots}^2 \rangle \\ &= \sum_k \sum_l \dots |\tilde{M}_c|^2 \langle n_{k,l,\dots} \rangle \\ &= N \int f_{\text{ens}}(t) dt \int \dots |\tilde{M}_c|^2 f_c(\alpha, \beta, \dots) d\alpha \dots \\ &= N |\tilde{M}_c|^2, \end{aligned}$$

• Note  $\langle dn_{k,l,\dots}^2 \rangle = \langle n_{k,l,\dots} \rangle$ , because  $n_{k,l,\dots}$  follows the Poisson distribution.

• Finally,

$$\begin{aligned} \langle |\tilde{M}_{\text{ens}}|^2 \rangle &= N^2 |\tilde{f}_{\text{ens}}|^2 (|\langle \tilde{M}_c \rangle|^2 + N |\overline{|\tilde{M}_c|^2}|) \\ &= N^2 |\langle \tilde{M}_c \rangle|^2 (|\tilde{f}_{\text{ens}}|^2 + \frac{1}{N}) + N (|\overline{|\tilde{M}_c|^2}| - |\langle \tilde{M}_c \rangle|^2) \\ &= N^2 |\langle \tilde{M}_c \rangle|^2 (|\tilde{f}_{\text{ens}}|^2 + \frac{1}{N}) + N \sigma_c^2, \end{aligned}$$

where  $\sigma_c^2 = (|\overline{|\tilde{M}_c|^2}| - |\langle \tilde{M}_c \rangle|^2)$  is introduced, representing the difference between the average ESD of the component discharge and the ESD of the averaged component discharge. If the component discharges are identical,  $N\sigma_c^2 = 0$  and

$$\langle |\tilde{M}_{\text{ens}}|^2 \rangle = N^2 |\langle \tilde{M}_c \rangle|^2 (|\tilde{f}_{\text{ens}}|^2 + \frac{1}{N}).$$

## Simulation Results

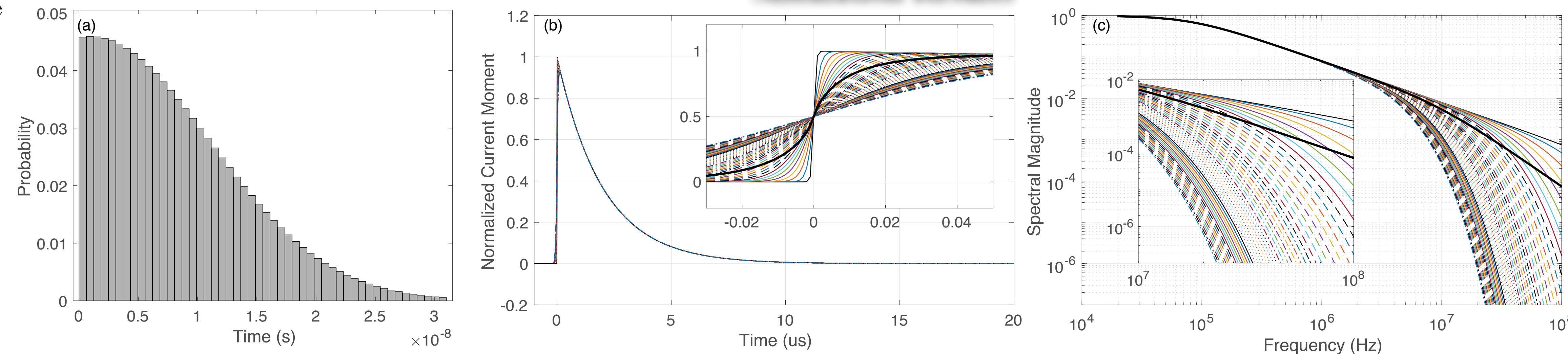


Figure above shows the properties of assumed streamer current pulses. (a) Probability of the streamer current rise time. (b) The streamer current pulse for each rise time. The inset shows a zoom-in view of the fast rise of the current, and the thick black curve is the average current with the probability distribution from (a). (c) The spectral magnitude of each current pulse in (b) and their mean (thick black curve).

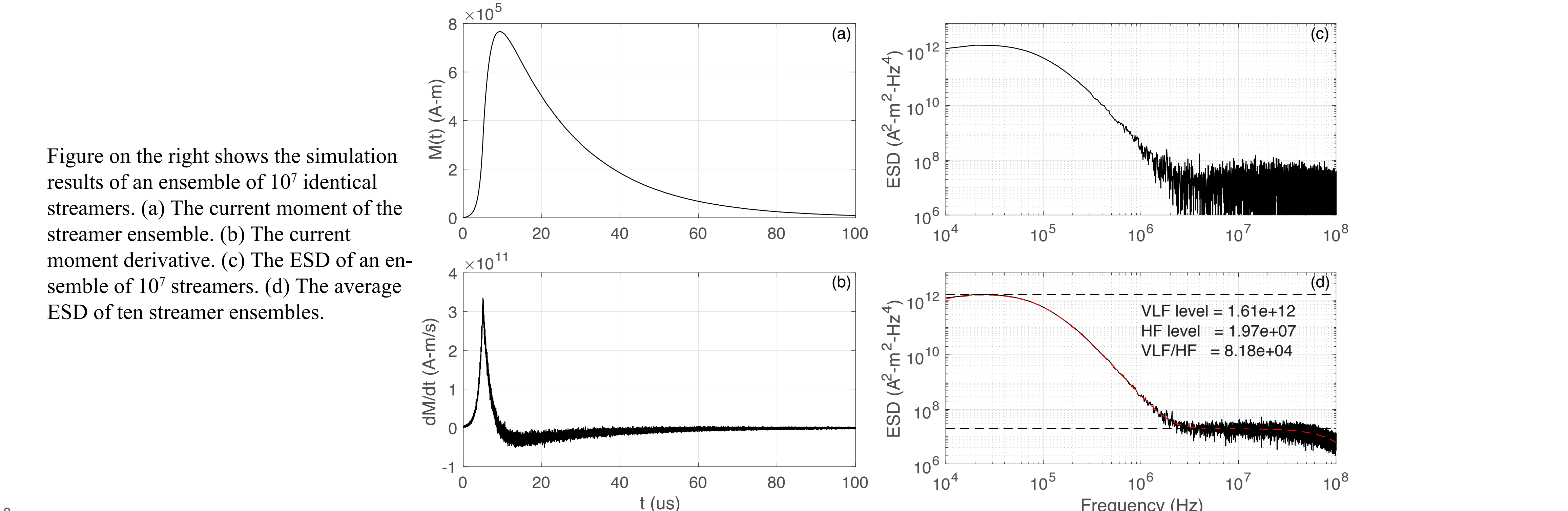


Figure on the right shows the simulation results of an ensemble of  $10^7$  identical streamers. (a) The current moment of the streamer ensemble. (b) The current moment derivative. (c) The ESD of an ensemble of  $10^7$  streamers. (d) The average ESD of ten streamer ensembles.

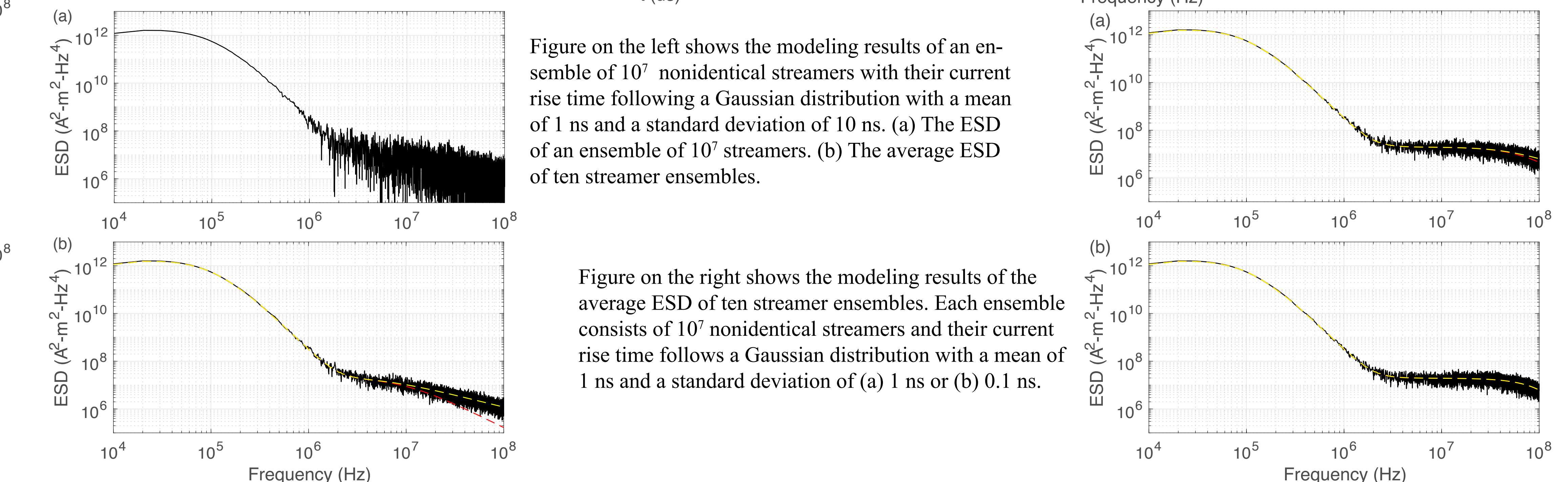


Figure on the left shows the modeling results of an ensemble of  $10^7$  nonidentical streamers with their current rise time following a Gaussian distribution with a mean of 1 ns and a standard deviation of 10 ns. (a) The ESD of an ensemble of  $10^7$  streamers. (b) The average ESD of ten streamer ensembles.

Figure on the right shows the modeling results of the average ESD of ten streamer ensembles. Each ensemble consists of  $10^7$  nonidentical streamers and their current rise time follows a Gaussian distribution with a mean of 1 ns and a standard deviation of (a) 1 ns or (b) 0.1 ns.

## Summary and Conclusion

A theory has been developed to calculate the electrical properties of a system of nonidentical streamers. In the context of NBEs, the rise time of the streamer current is much shorter than the NBE duration, but the fall time can be comparable. As a result, the low frequency part of the radio spectrum of the NBE is determined by coherent summation of the radio emissions of the individual streamers, while the high frequency part by incoherent summation. The spectral magnitude ratio between the two spectral regions is therefore proportional to the number of streamers in the event. The variations among the timescales of the streamers can result in the enhancements of the spectral magnitudes in the frequency ranges that correspond to the variations. For example, if the standard deviation of the rise time of the streamers is 10 nanoseconds, the radio emissions above a few tens of MHz are enhanced.

## Acknowledgments

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