Parametric Decay of Circularly Polarized Alfven Wave: One Dimensional and Multidimensional Simulation

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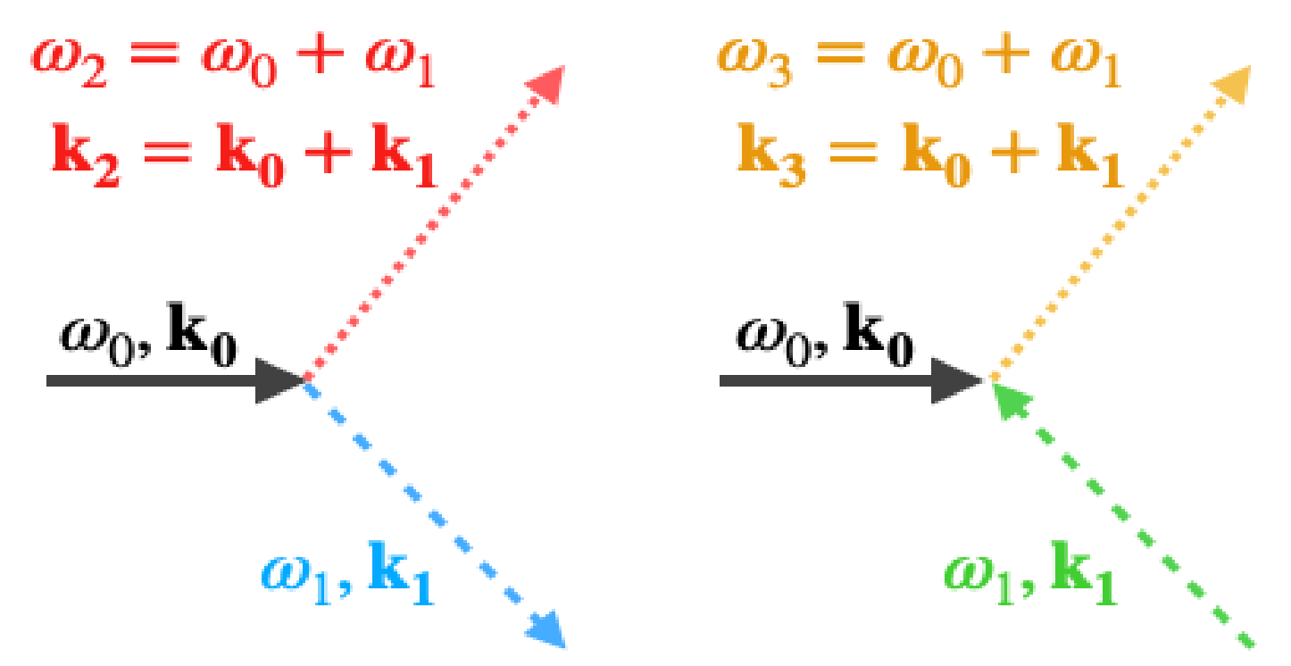
Introduction

In the solar corona and solar wind, Alfvén waves transport magnetic energy [1]. In this region, Alfvén waves have large amplitudes, thus nonlinear wave–wave and wave–particle interactions are expected to be important.

For instance, in compressible plasmas, large amplitude Alfvén waves are subject to the parametric decay instability (PDI), where a forward-propagating Alfvén wave (pump wave) decays into a backward-propagating Alfvén wave and a forward-propagating ion acoustic, fast, or slow wave.



The diagram above shows the decay of the pump wave into a low-frequency wave and a down-shifted high-frequency wave (Left) and the scattering of the pump wave by a low-frequency wave, exciting an up-shifted high-frequency wave (Right).

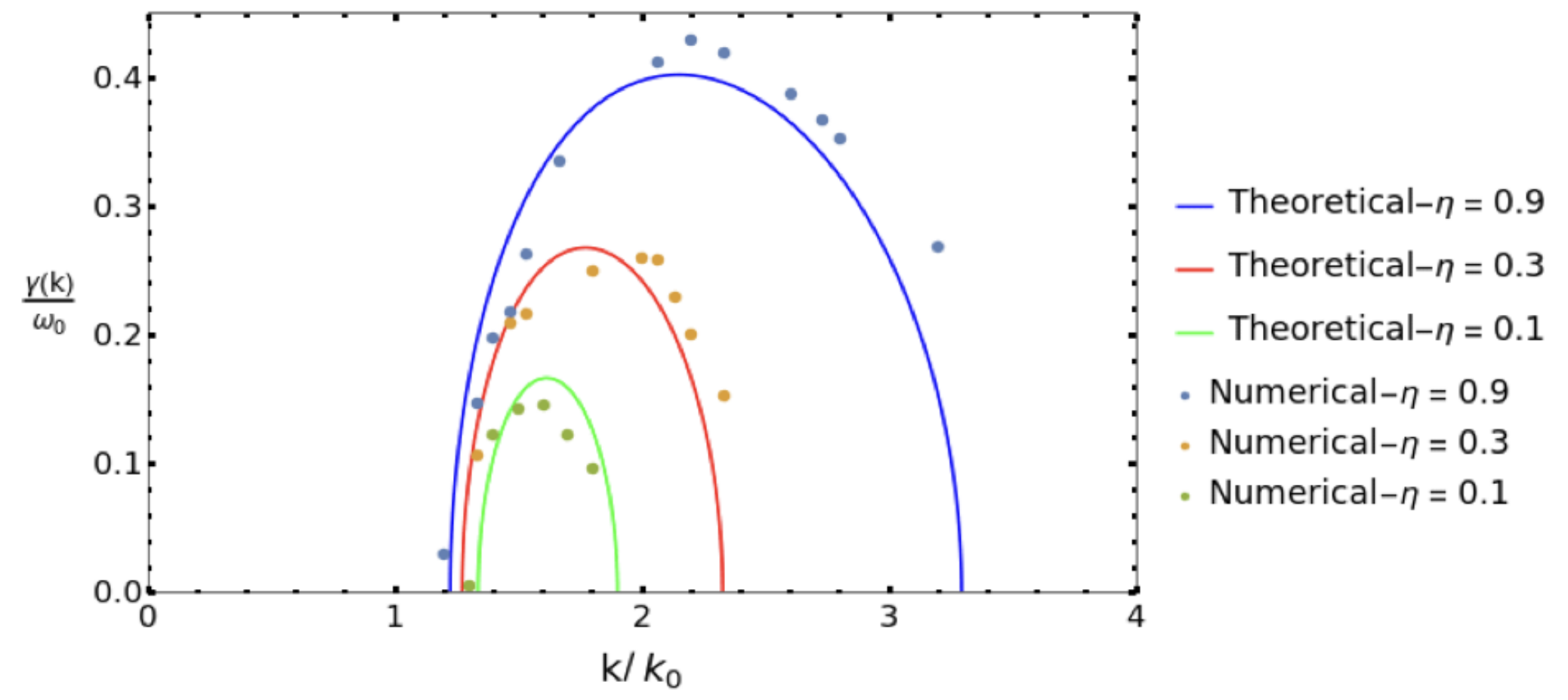
Here, the evolution of the parametric decay of a high amplitude Alfvén wave is investigated using the Magnetic Reconnection Code (MRC) [2,3]. The present work benchmarks progress towards nonlinear simulations of 3D parametric decay in the solar wind, including effects of Landau damping.

Theory

The low-frequency behavior of a magnetized plasma is represented by the set of ideal MHD equations for which a large-amplitude, circularly polarized wave is an exact solution. With a linear perturbation, the dispersion relation governing instabilities [4] is,

$$(\omega^2 - \beta k^2)(\omega - k)[(\omega + k)^2 - 4] = \eta^2 k^2(\omega^3 - k\omega^2 - 3\omega + k) \qquad (1)$$

where ω and k are normalized by the frequency and wavenumber of the pump Alfvén wave, $\eta = \delta B/B_0$, and $\beta = C_s^2/v_A^2$. Instabilities occur when, for real values of k, the dispersion relation yields complex roots for $\omega = \omega_r + i\gamma$ with $\gamma > 0$.



We performed 1-D ideal MHD simulations to find the magnetic growth rates. Plots of theoretical and numerical values of $\gamma(k)$ versus k for $\beta = 0.1$ and three different values of η are shown.

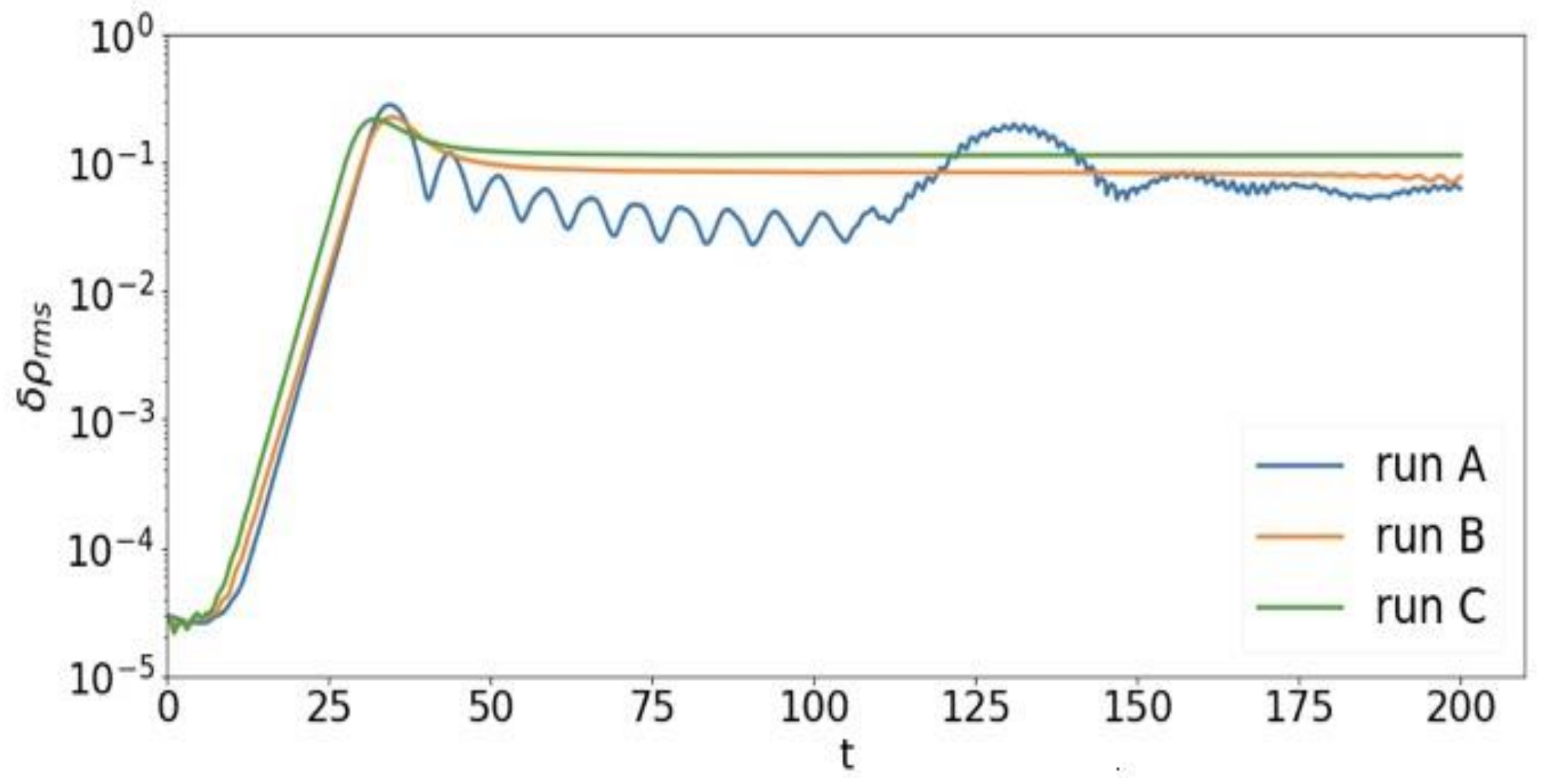
1-D Results:

We performed a series of simulations to reproduce the results of del Zanna et al. [5] as a validation of our methodology. We consider an initial state with a uniform magnetized plasma ($\boldsymbol{B_z} = B_0$; $V_0 = 0$; $\rho = \rho_0$; $p = p_0 = (\beta / \gamma) B_0^2$) plus a circularly polarized monochromatic Alfven wave in parallel propagation. Results about the long-term nonlinear evolution of PDI in 1-D are listed for three cases below. A resolution of N = 2048 grid points was used for each case.

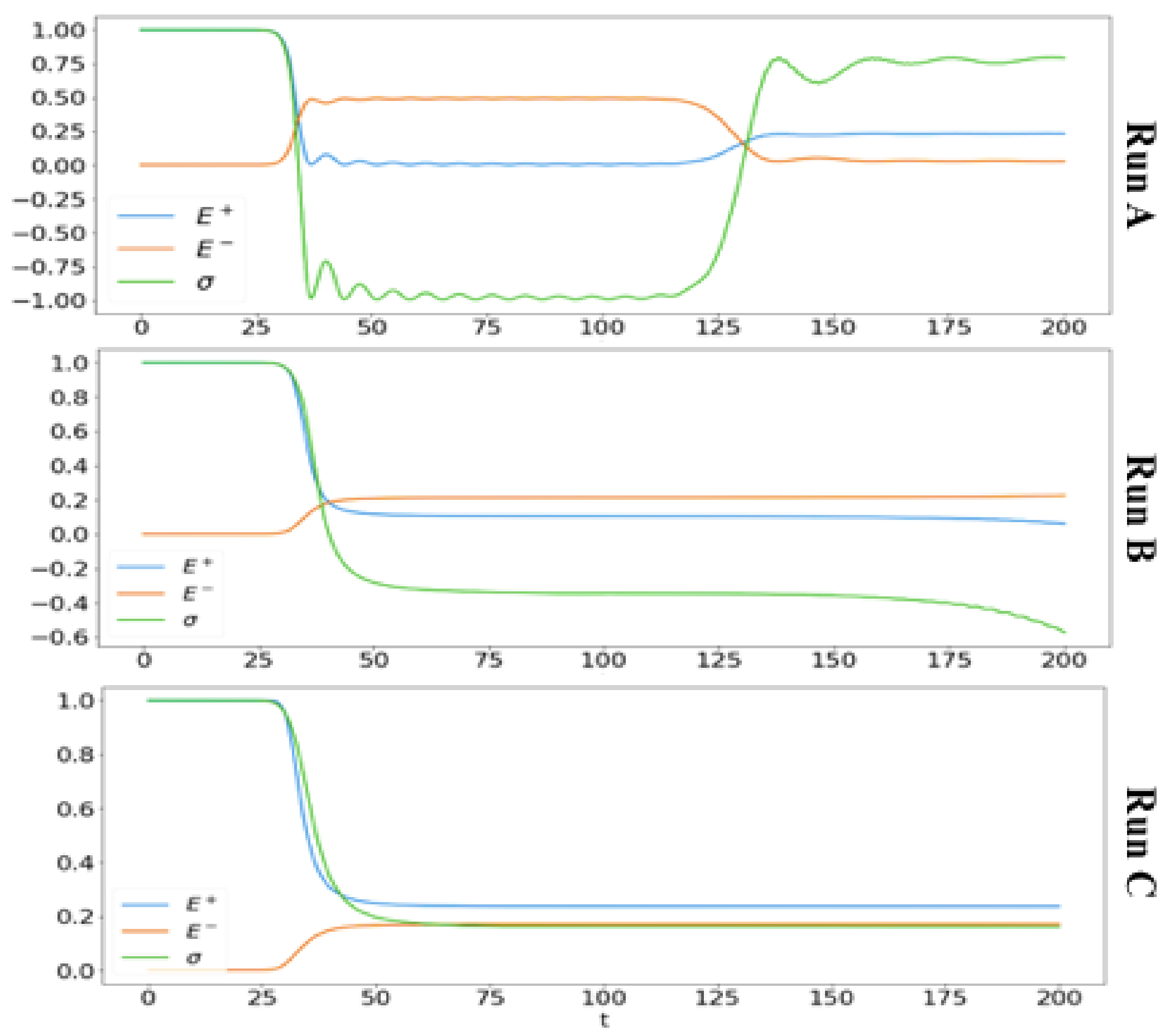
Run	η	β	k_0	k_c	k_t^+	k_t^-	$\gamma_{Theoretical}$	$\gamma_{Numerical}$
A	0.2	0.1	4	6	10	2	0.41	0.40033
B	0.5	0.5	4	5	9	1	0.39	0.38666
C	1.0	1.2	4	5	9	1	0.41	0.42311

The linear growth rates derived above match those predicted by theory within an error less than 4%.

The rms density fluctuations, $\delta\rho_{rms}$, for all three cases shows exponential growth and subsequent saturation. The compressive modes reach the greatest amplitude when saturation occurs, corresponding with the maximum of $\delta\rho_{rms}$.



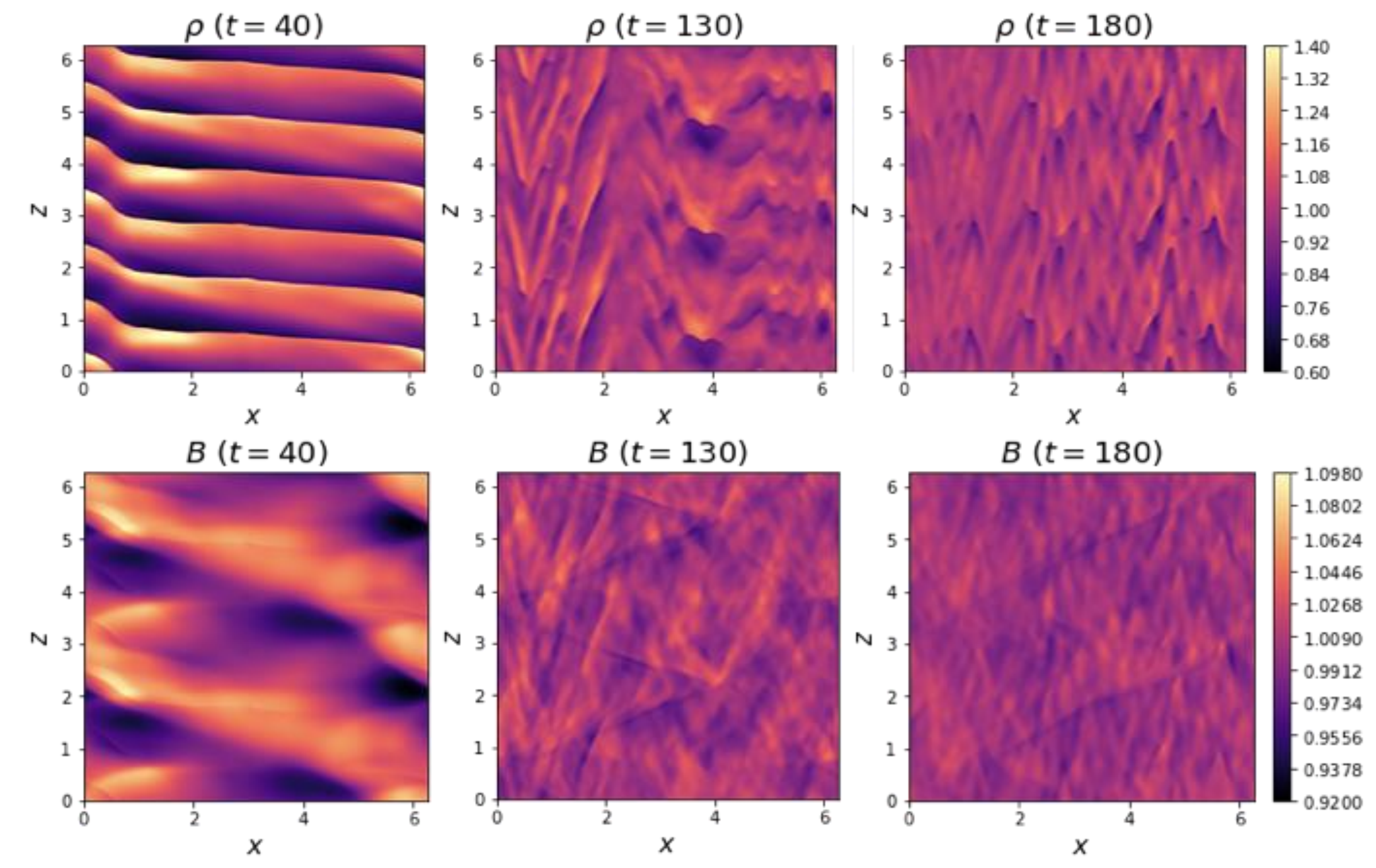
The figure below shows the temporal evolution of the cross helicity σ and Elsasser variables $E^{\pm}$. At the time of nonlinear saturation, σ drops with E^+ while E^- grows in time, corresponding to the backscattered Alfven wave with wave number k_{t-}. The secondary instability occurs at time $t \simeq 130$ in run A. The sign of σ after the first instability strongly depends on the β.



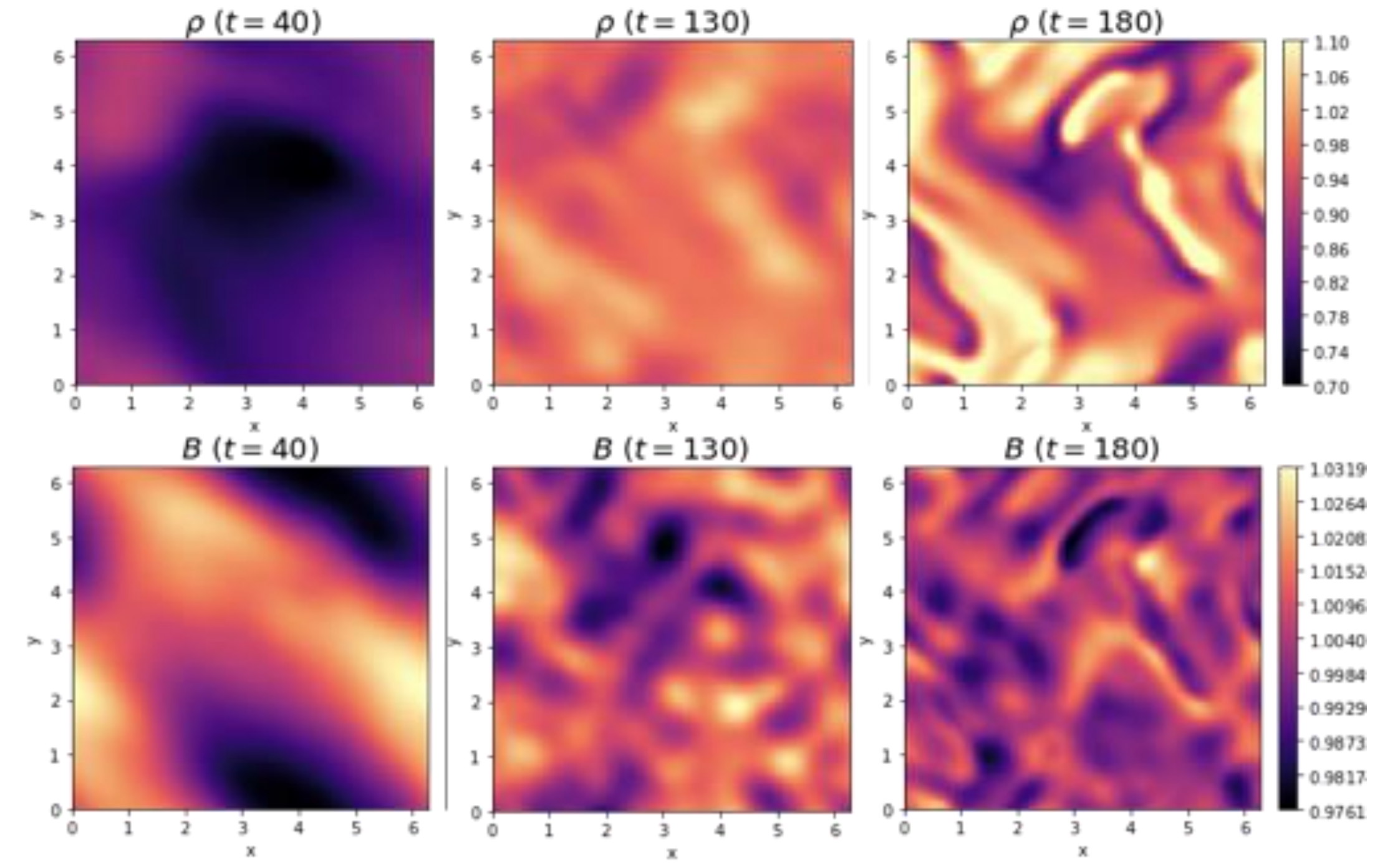
2-D and 3-D Results

Moving to higher dimensions, the same initial conditions are kept, but point-wise random noise is introduced in the whole volume. Following, we limit our analysis to run A, reducing the resolution to 512^2 grid points in 2-D and to 128^3 in 3-D.

Snapshots of density and magnetic field strength are presented for 2-D. The slow magnetosonic shock fronts are initially parallel to $\mathbf{B_0}$ with a modulation in the perpendicular direction which grows in time. In the parallel direction, the inverse cascade $k_{zc} = 6 \longrightarrow 3$ after the secondary decay is observed.



Since parametric decay is essentially a 1-D process, the longitudinal behavior is practically identical in 2-D and 3-D. Slices at the injection boundry ($z = 0$) for density and magnetic field strength at three different times are shown below for 3-D simulations.



Note the primary and secondary decays occur also in 2-D and 3-D though the occurrence time is slightly delayed in higher dimensions (the secondary decay in 3-D occurs much later).

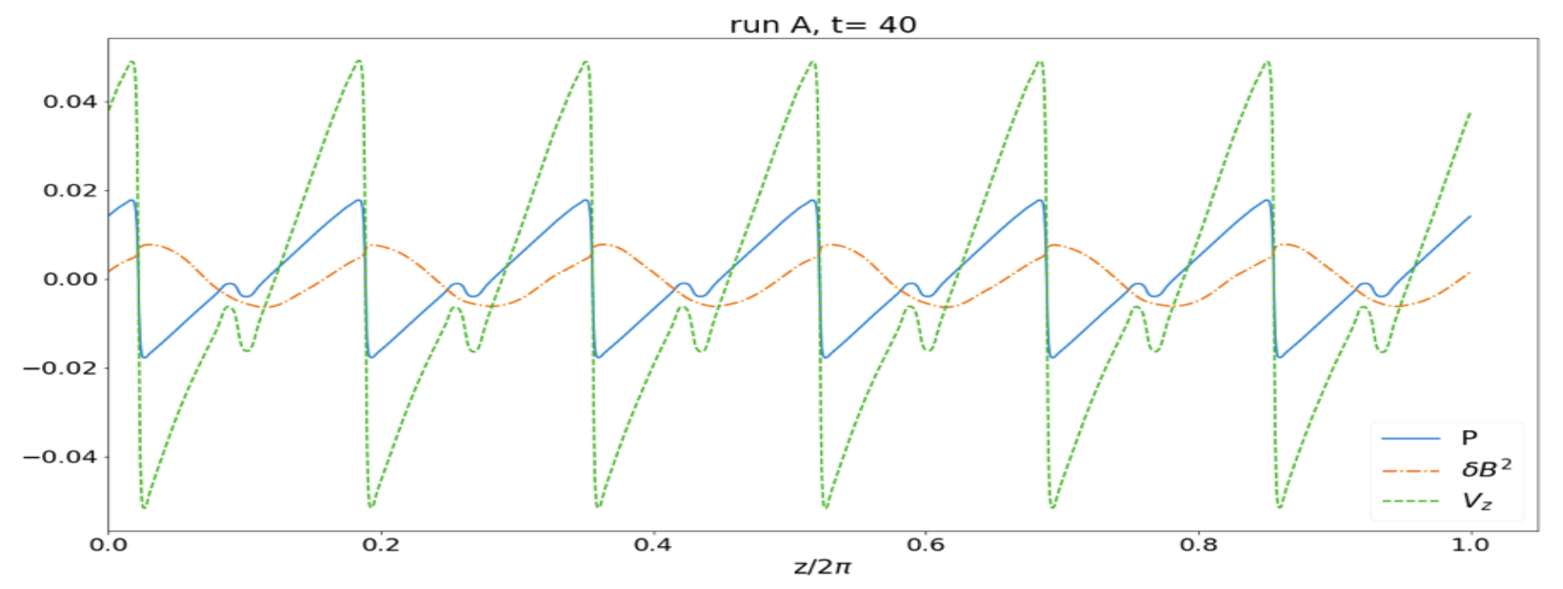
Magnetic Reconnection Code (MRC)

The Magnetic Reconnection Code (MRC) is a fully compressible extended MHD code (resistive MHD is a special case). It features a generalized Ohm's Law including the Hall term and electron pressure gradient and has options which enable studies of free and forced reconnection. The code is parallel and implements options for discretization of the extended MHD equations.

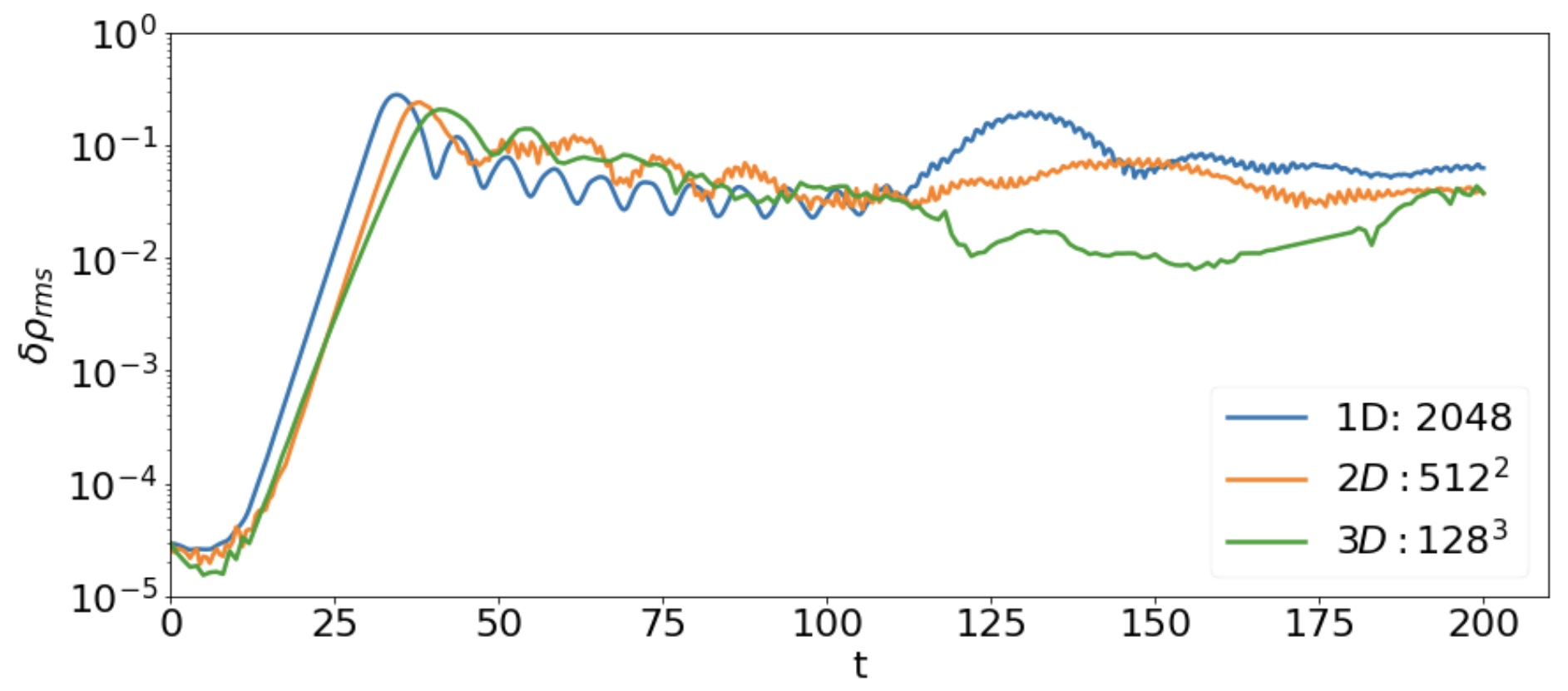
Summary and Future Work

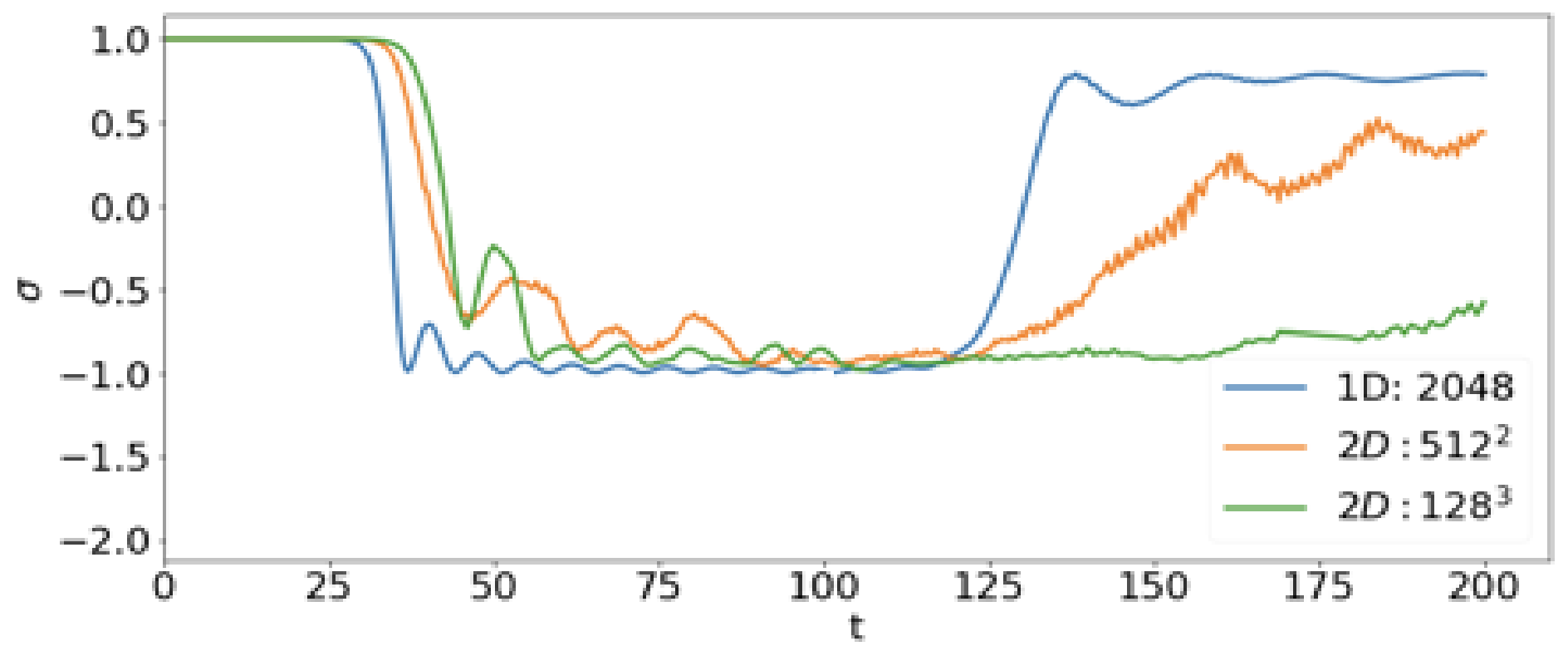
Key Points:

- After a linear growth phase, the instability saturates when the generated compressive mode steepens into a train of shocks.



- As in the work of del Zanna et al., The value of the cross helicity is dependent on the plasma beta. As β increases, the relative drop in cross helicity decreases as well as $E+$. When $\beta < 1$, σ becomes negative after saturation; when $\beta > 1$, σ remains positive; when $\beta \simeq 1$, σ tends towards zero.
- Similar also to del Zanna et al., Secondary instabilities followed the first decay for low-β. Through this inverse cascade, the pump wave's energy is reduced to zero at saturation and cross helicity reverses, switching from $\sigma \approx +1$ to $\sigma \approx -1$ at each saturation step.
- For each instability occurrence, the total energy in the Alfvenic modes is reduced, roughly, by a factor of two.





Future Work:

- Parametric Decay Instability provides a mechanism to dissipate wave energy into plasma through Landau damping of ion acoustic waves. We will use ideal MHD simulations which include the Landau damping effect to investigate PDI further.

References

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