Chaos in the Solar Cycle: using data to drive predictions

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Introduction

– Our sun is a G–class magnetic star that undergoes magnetic pole reversals with an average period of approximately 22 years, and an activity cycle with half that period.

– During peaks in activity, the solar surface is populated by bipolar active regions, like the lighter regions shown in the image below:

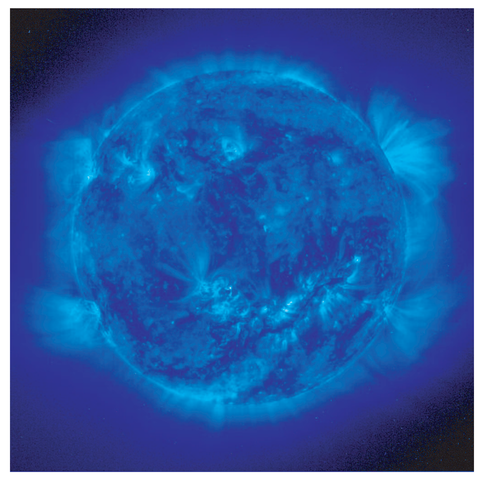


Figure 1: *Active regions and magnetic loops imaged in the Fe IX/X 171Å line by NASA's Solar and Heliospheric Observatory (SOHO). Image credit:* NASA.

– These active regions are co–located with sunspots, which appear as dark regions when the solar surface is viewed in the visible spectrum.

– The following image demonstrates the spatial scale of a typical sunspot in comparison to the Earth:

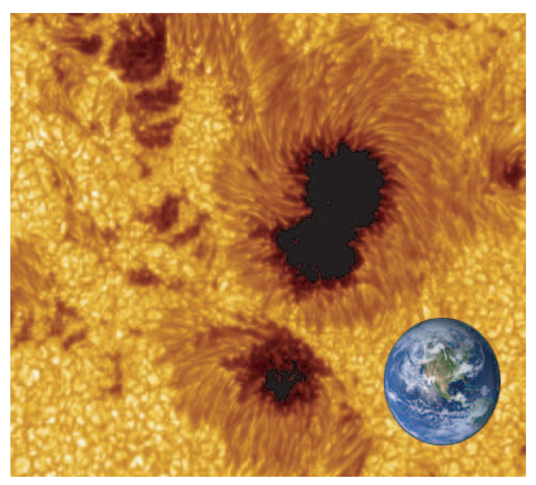


Figure 2: *Comparison of the size of the Earth to a typical sunspot. Image credit:* Windows to the Universe.

– Sunspots have been observed since 325 BC, and reasonably reliable data extend back to 1749 AD.

– Edward Maunder established an intuitive way to look at the motion of sunspots over the solar surface during one or more activity cycles when he introduced the "butterfly diagram" in 1904:

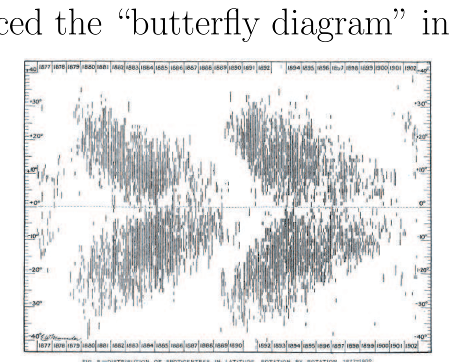


Figure 3: *E. Maunder's original "butterfly diagram" ([3]).*

– Since sunspots are co–located with active regions, their numbers mirror the rise and fall of solar activity.

– Certain solar events, such as solar flares and coronal mass ejections, pose threats of varying severity to life on Earth.

– The data are chaotic, so linear analysis techniques (e.g. Fourier decomposition) are not valid.

Chaotic Time Series Analysis

A time series $\mathbf{x}(t)$ with nonlinear behavior can be reconstructed as an N–dimensional phase space vector $\mathbf{y}(t)$ using the method of time delays [1].

$$\mathbf{y}(t) = (\mathbf{x}(t), \mathbf{x}(t+\tau), ..., \mathbf{x}(t+(N-1)\tau))$$

Determining the correct dimension (N) and the correct time delay (τ) to implement are not straightforward tasks.

If we *can* reconstruct $\mathbf{y}(t)$, we can ask whether trajectories of initial conditions separated by an arbitrarily small distance (δ_0) converge or diverge exponentially along the j^{th} component of $\mathbf{y}(t)$: $\delta(t) \sim \delta_0 e^{\lambda_j t}$ [7]. The λ_j ($0 \le j \le N$) are the Lyapunov exponents. Chaotic systems exhibit sensitive dependence on initial conditions, so that nearby initial condition diverge exponentially (i.e. there is at least one j such that $\lambda_j > 0$).

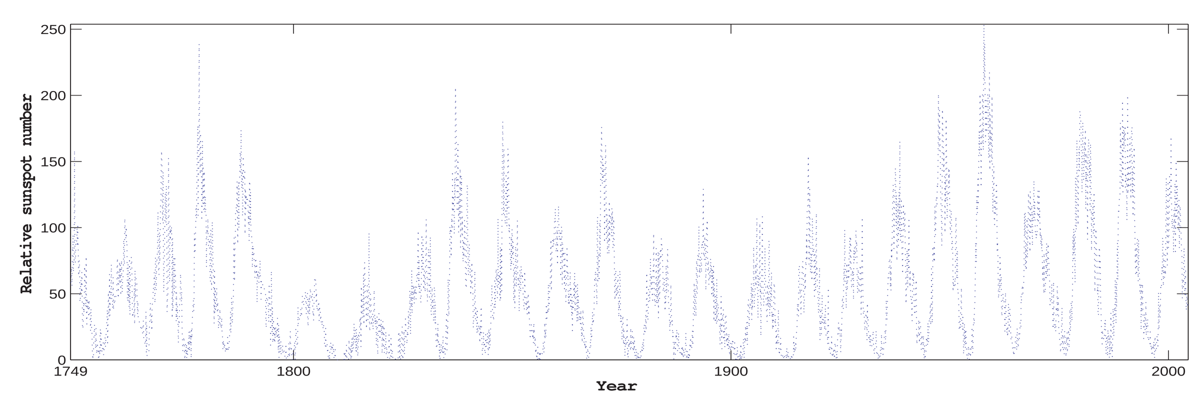
We employed neural network algorithms *chaosfn.m* and *chaostest.m*, available at the Matlab file exchange (`http://www.mathworks.com/matlabcentral/fileexchange/`)

Chaos in Sunspot Number

Relative sunspot number, established by R. Wolf ca. 1850. (*Available at* `http://solarscience.msfc.nasa.gov/SunspotCycle.shtml`)

$$R = k(10g + s)$$

g = number of groups of spots s = number of individual spots k = scale factor for inter–observatory comparison.

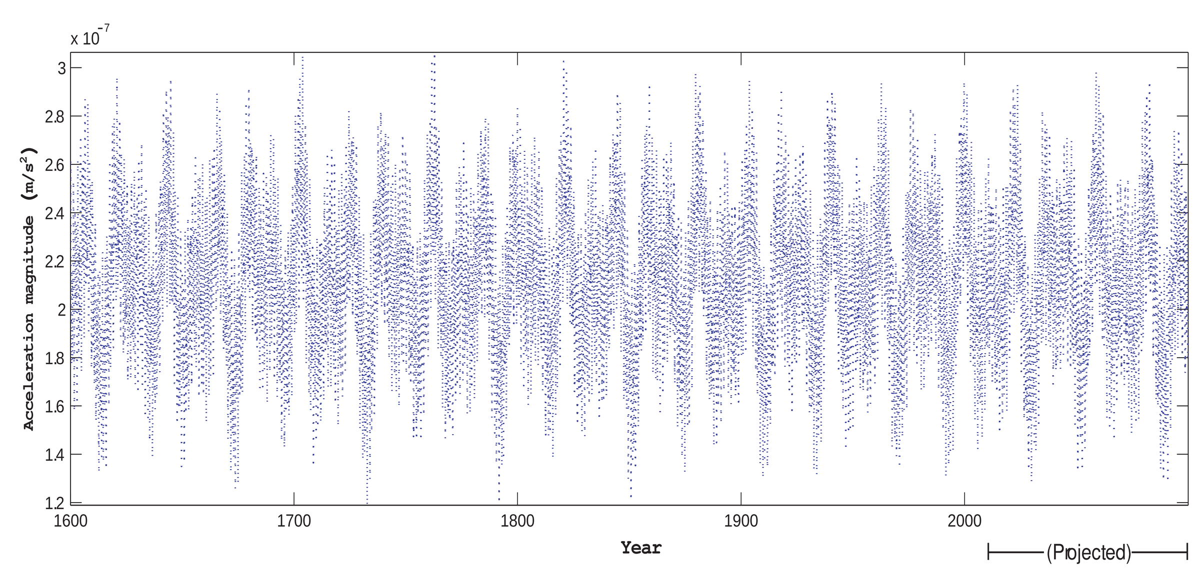


Maximum Lyapunov exponent is $\lambda \cong 0.056$, slightly higher than that found by [4], who used smoothed data.

Chaos in Solar Acceleration

Magnitude of solar acceleration about the barycenter of the solar system, derived from NASA Jet Propulsion Laboratory ephemerides. (*Available at* `http://ssd.jpl.nasa.gov/?horizons`)

$$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$



Maximum Lyapunov exponent is $\lambda \cong 0.079$, indicating chaos.

Planetary Tides?

Compromise frequency of two non–linear oscillators with frequencies (ω_1, ω_2) and coupling constants (k_1, k_2) ([7]):

$$\omega^* \equiv \frac{k_1\omega_2 + k_2\omega_1}{k_1 + k_2}$$

Compromise period of Jupiter and Saturn (with each planet's mass used as a coupling constant):

$$\tau^* = \left(\frac{m_J + m_S}{\tau_J m_J + \tau_S m_S}\right) m_J m_S \approx 22.0\ yr$$

Possible scenerio: The orbital motion of Jupiter and Saturn couples (perhaps weakly and with dissipation) to the internal dynamics of the Sun enough to give solar activity a kick when it is most sensitive to chaotic dynamics.

The similarity is encouraging, but there are more critics (e.g. [6] and [2]) of planetary forcing than there are advocates (e.g. [5]), and we must remain skeptical at this point.

First Conclusions

We have...

– confirmed a previous conclusion of chaos in the sunspot cycle [4], using publicly available Matlab code.

– found that the magnitude of the Sun's acceleration about the solar system's barycenter, and thus the force on the solar center of mass, is chaotic.

– found that Jupiter and Saturn orbit the barycenter of the solar system with a "compromise period" equal to the average period of solar magnetic pole oscillations.

Open Questions

– How (if at all) is the chaos in the Sun's acceleration related to chaos in the sunspot number?

– Are similarities in the orbital periods of planets and the average length of sunspot cycles coincidence or can planetary tides affect the solar cycle in a nonlinear way?

– Can we use phase space reconstructions of chaotic solar motion to reliably predict solar activity on time scales long enough to be useful?

References

[1] Henry D. I. Abarbanel. *Analysis of Observed Chaotic Data.* Springer: New York, 1996.

[2] D. K. Callebaut, C. de Jager, and S. Duhau. The influence of planetary attractions on the solar tacholine. *J. Atmos. Sol.–Terr. Phys.*, 2012.

[3] E. Walter Maunder. Note on the Distribution of Sun–spots in Heliographic Latitude, 1874–1902. *Mon. Not. R. Astron. Soc.*, 64, 1904.

[4] Michael D. Mundt, II Maguire, W. Bruce, and Robert R. P. Chase. Chaos in the Sunspot Cycle: Analysis and Prediction. *J. Geophys. Res.*, 96(A2):1705–1716, 1991.

[5] Nicola Scafetta. Multi–scale harmonic model for solar and climate cyclical variation throughout the Holocene based on Jupiter–Saturn tidal frequencies plus the 11–year solar dynamo. *J. Atmos. Sol.–Terr. Phys.*, 2012.

[6] Charles M. Smythe and John A. Eddy. Planetary tides during the Maunder Sunspot Minimum. *Nature*, 1977.

[7] Steven H. Strogatz. *Nonlinear Dynamics and Chaos.* Westview Press, 1994.