On the physical dimension of the turbulent sublayer at the turbulent/non-turbulent interface

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Entrainment & the turbulent/non-turbulent interface

- Entrainment is a multi-scale process facilitated by the turbulent/non-turbulent interface (TNTI)
- The TNTI is an intense shear layer composed of the viscous superlayer (VS) and the turbulent sublayer (TS)

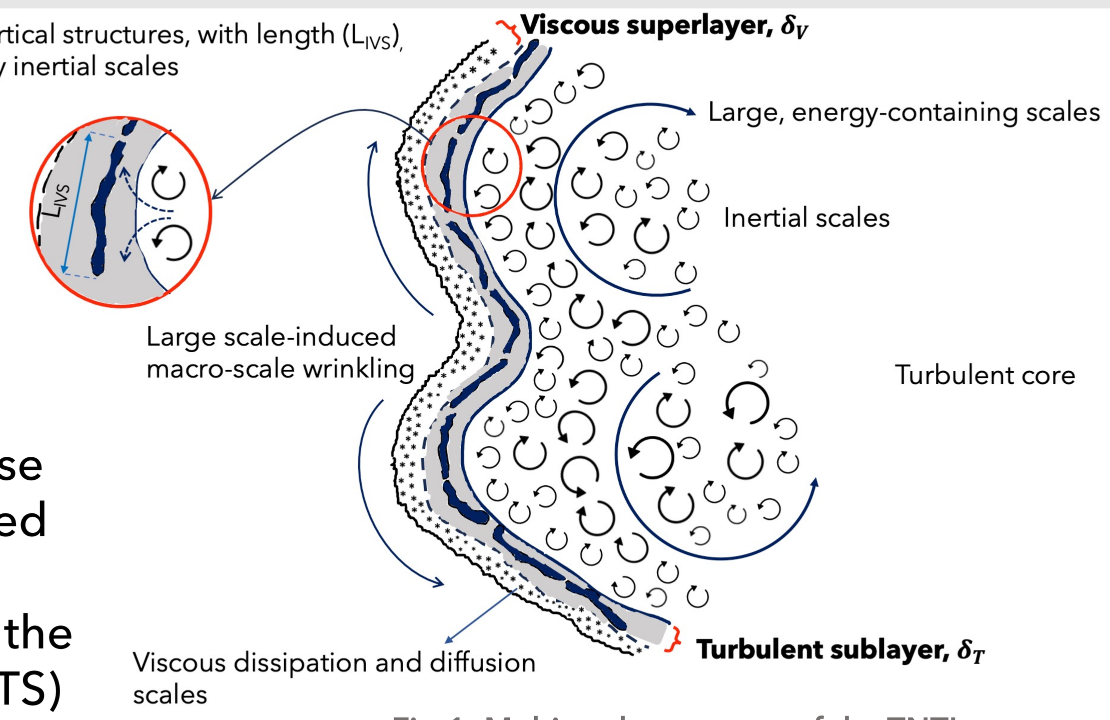


Fig.1: Multi-scale structure of the TNTI

- The TNTI can be described using long-time dynamics, relating to global features, and short-time dynamics, which determine local features, such as its thickness δ_T
- Previous studies [1,2] show that $\delta_T \sim O(10\eta)$, indicating that the short-time dynamics of the TNTI are driven by the Kolmogorov scales. Such scales are challenging to resolve in experiments exploring TNTI dynamics at high Reynolds number, Re_λ

This work leverages the short-time dynamics of TNTI to:

- Characterize δ_T by a length scale closely matching its physical dimension to relax the resolution required in experiments
- Explore the physical implications of such a length scale

Entrainment: A problem of propagating TNTI

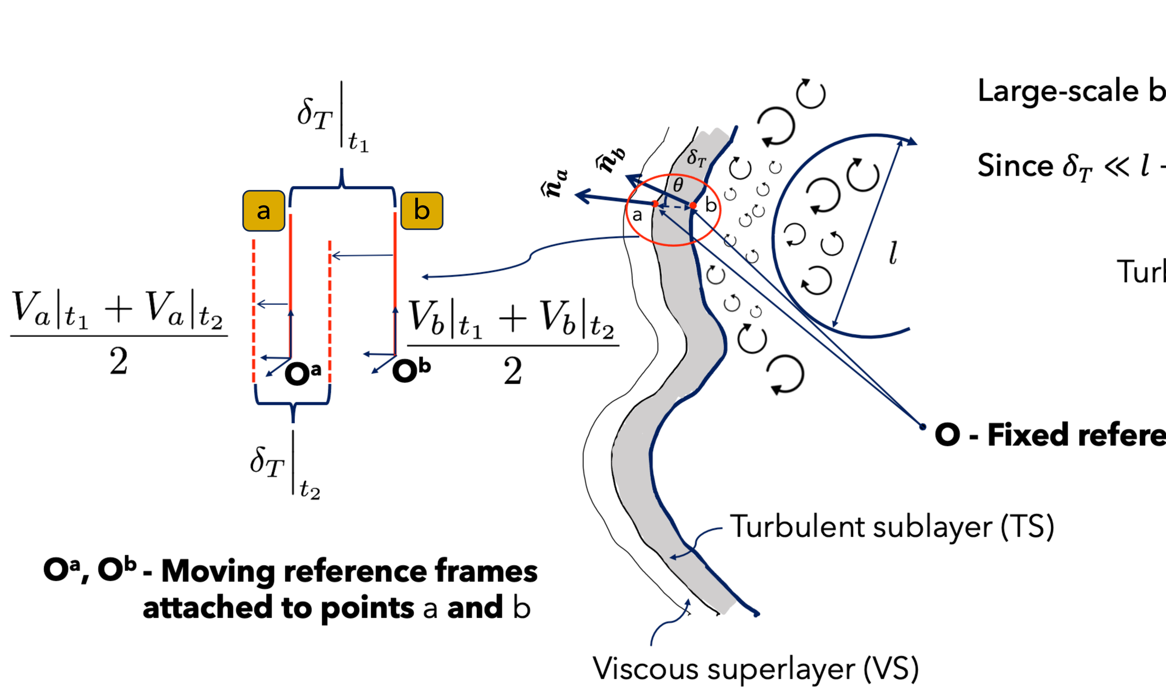


Fig. 2: Schematic depiction of a propagating TNTI; inset depicts TS motion at two successive instants in time.

- TS motion is due to large-scale advective motions in reference frame O and the propagating motion in reference frames **Oᵃ and Oᵇ**
- Motion of interest is in reference frames **Oᵃ** and **Oᵇ** by considering edges "a" and "b" as vorticity iso-surfaces [3]

$$v_p\hat{\mathbf{n}} = \boxed{\frac{2\omega_i\omega_j s_{ij}}{|\nabla\omega^2|}}\hat{\mathbf{n}} + \frac{2\upsilon\omega_i\nabla^2\omega_i}{|\nabla\omega^2|}\hat{\mathbf{n}} = \boxed{v_\alpha}\hat{\mathbf{n}} + v_\upsilon\hat{\mathbf{n}}, \qquad \hat{\mathbf{n}}_\mathbf{a} = \frac{\nabla\omega_a^2}{|\nabla\omega_a^2|}, \qquad \hat{\mathbf{n}}_\mathbf{b} = \frac{\nabla\omega_b^2}{|\nabla\omega_b^2|}. \quad (1)$$

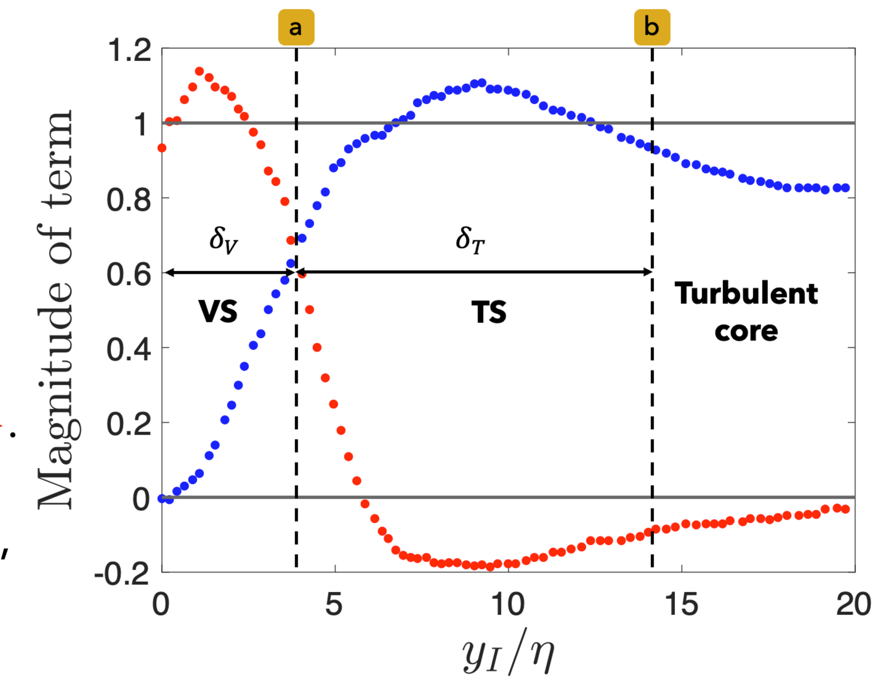
- Considering motions in reference frames O (see inset in Figure 2):

$$\lim_{\Delta t \to 0}\left\{\frac{\delta_T|_{t_1} - \delta_T|_{t_2}}{\Delta t} \cong \frac{v_p^b|_{t_1} + v_p^b|_{t_2}}{2} - \frac{v_p^a|_{t_1} + v_p^a|_{t_2}}{2}\right\} \longrightarrow \delta_T = \int_0^{t_I}(v_p^b - v_p^a)dt \sim \boxed{vt_I}. \quad (2)$$

Scaling δ_T requires scaling v and t_I ; v is a to-be-determined velocity scale and t_I is the characteristic time scale over which the TS travels a distance equal to its instantaneous thickness

TNTI & enstrophy: An estimate for t_I

Fig. 3: Data [1] showing the balance of enstrophy as a ratio of the viscous diffusion to viscous dissipation (in red) and as a ratio of inviscid vortex stretching to viscous dissipation (in blue) across the entire TNTI. For labels "a" and "b", refer to Fig. 2.



Within the TS:

$$\omega_i\omega_j s_{ij} \approx \nu\left(\frac{\partial\omega_i}{\partial x_j}\frac{\partial\omega_i}{\partial x_j}\right) + \frac{\nu}{2}\frac{\partial^2(\omega_i\omega_i)}{\partial x_j^2}.$$

By order of magnitude comparison,

$$s_{ij} \sim \nu(\partial_x)^2 + \nu\partial_x^2$$

Spatial gradients inversely related to δ_T

$t_I \sim \frac{\delta_T^2}{\nu}$ – Characteristic time scale for the short-time dynamics of the TS

Turbulence cascade: A means for estimating v

Average description of turbulence cascade is due to interacting scales of motion having similar time scales ⟶ $t_E \sim t_I$ ⟶ $u'\left(\frac{s}{l}\right)^{1/3} \sim \frac{\delta_T^2}{\nu}$

Time scale matching allows for estimating a suitable choice for the scale s by comparing its order of magnitude with l, λ and η

- Comparing the scale s with η, the Kolmogorov length scale ⟶ $\frac{s^{2/3}l^{1/3}}{u'}\frac{\nu}{\eta^2} \sim \frac{\delta_T^2}{\eta^2} \approx K'^{2/3}$ ⟶ $\frac{s}{\eta} \sim K' \sim 10^3$ [4]
- Comparing scale s with the integral length scales (l) violates the assumption that $\delta_T \ll l$; see Figure 2
- Comparing the scale s with λ, the Taylor length scale ⟶ $\frac{s^{2/3}l^{1/3}}{u'}\frac{\nu}{\lambda^2} \sim \frac{\delta_T^2}{\lambda^2}$ ⟶ $\frac{s}{\lambda} \sim Re_\lambda\frac{\delta_T^3}{\lambda^3}$

Strain-inducing scales of motion s are not associated with dissipative motions but instead contribute to v_α - see Eq. (1)

$$\frac{\omega_i\omega_j s_{ij}}{|\nabla\omega^2|} = v_{\alpha,p}^b \sim \left(\frac{u'}{\lambda^{2/3}l^{1/3}}\right)\delta_T Re_\lambda^{2/9}$$

Viscous diffusion acts as vorticity sink (see red for point labelled b in Fig. 3); negatively contributes to v_p^b:

$$v_p^b - v_p^a = v_{\alpha,p}^b - (v_p^a + v_{\nu,p}^b) \sim v \sim \left(\frac{u'}{\lambda^{2/3}l^{1/3}}\right)\delta_T Re_\lambda^{2/9} - v'_\eta.$$

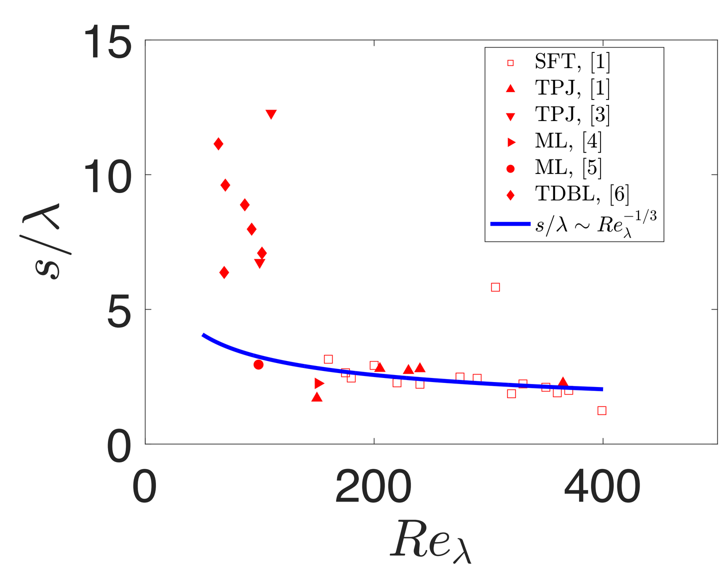


Fig. 4: Data from literature with varying turbulence forcing mechanisms confirming that $s \sim \lambda$. (See Fig. legend; SFT: shear free turbulence; TPJ: temporal planar jet; ML: Mixing layers; TDTBL: Temporally developing turbulent boundary layer).

Result: A mixed length-scale parameter for δ_T

Using Eq. (2),

$$vt_I \sim \left(\left(\frac{u'}{\lambda^{2/3}l^{1/3}}\right)\delta_T Re_\lambda^{2/9} - v'_\eta\right)\frac{\delta_T^2}{\nu} \longrightarrow \boxed{\frac{\delta_T}{\eta^{2/3}\lambda^{1/3}}} \sim Re_\lambda^{-1/9}\sqrt{1 + \boxed{\frac{\delta_T}{\eta}}} \approx 3$$

The two length scales are quadratically related and almost independent of Re_λ

δ_T/η vs $\delta_T/(\eta^{2/3}\lambda^{1/3})$: Statistical comparison

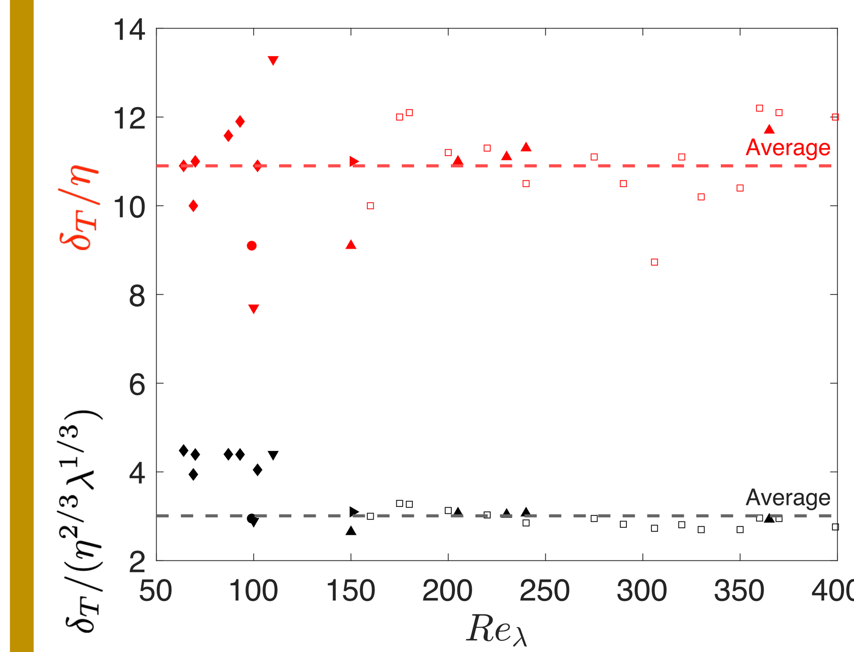


Fig. 5: Data normalized with the mixed-length scale parameter (in black) compared with the scaling parameter η (in red). Refer to legend in Figure 4 for flow type.

For $Re_\lambda \leq 400$

- $\eta^{2/3}\lambda^{1/3}$ closely matches the physical dimension of the TS

For $Re_\lambda < 100$

- Larger deviations exist due to insufficient scale separation

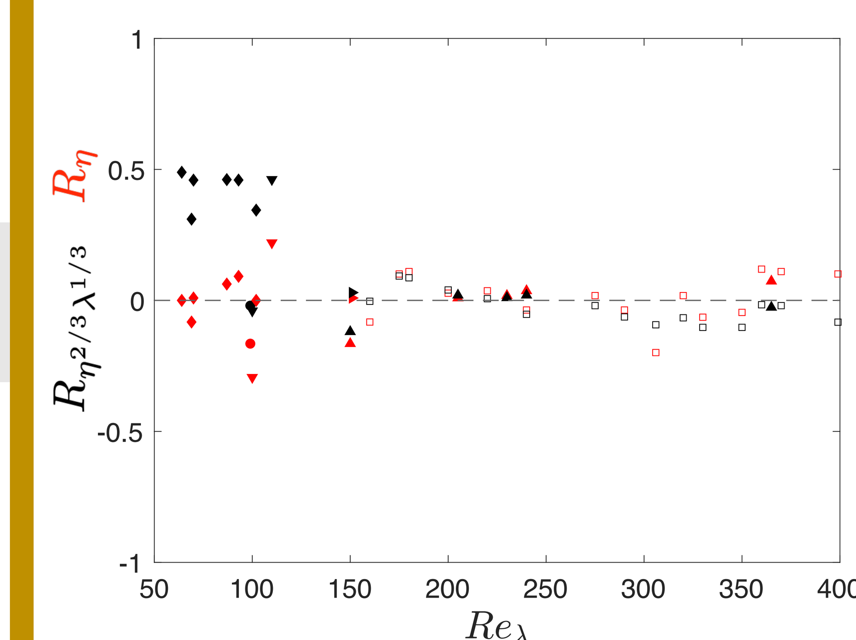


Fig. 6: Distribution for normalized residuals of the data points shown in Figure 5 for η (in red) and for $\eta^{2/3}\lambda^{1/3}$ (in black).

$$R_\eta = \frac{\delta_T/\eta}{\overline{\delta_T/\eta}} - 1 \quad \Big| \quad R_{\eta^{2/3}\lambda^{1/3}} = \frac{\delta_T/(\eta^{2/3}\lambda^{1/3})}{\overline{\delta_T/(\eta^{2/3}\lambda^{1/3})}} - 1.$$

Normalized residuals do not favor either parameter over the other for $100 < Re_\lambda < 400$

δ_T/η vs $\delta_T/(\eta^{2/3}\lambda^{1/3})$: Physics & Practice

δ_T/η assumes that strain-inducing scales of motion s correspond to η:

- Uses Burger's type vortex for explaining the TNTI dynamics
- Burger's vortex precludes capturing relevant flow physics at larger scales involving complex strain/vorticity interaction; $L_{IVS} \sim 100\,\eta$ [7]
- Scale dynamics due to η are seldom resolved in experiments

$\delta_T/(\eta^{2/3}\lambda^{1/3})$ uses strain-inducing scales of motion s corresponding to λ:

- Captures strain dynamics over relevant scales: $L_{IVS} \sim \lambda$ [7] (see Fig 1)
- Underscores the need for a physics-based scaling for TS and VS: viscoelastic flows show a different balance of operative time scales in VS and TS [8] (also see Fig 3)
- Acknowledges the inertially-dominated flow physics within the turbulent core through λ
- Identifies $\eta^{2/3}\lambda^{1/3}$ as a scale of dynamic importance and is more likely to be resolved in experiments

Acknowledgments & Citations

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