Implementing a PID Controller in a Balancing Beam System

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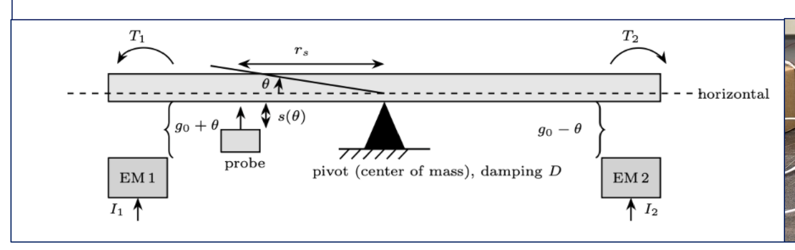
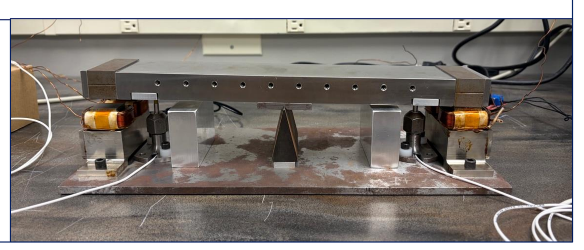
Introduction

The system consists of a rigid ferromagnetic beam that pivots about its center of mass, suspended at each end by opposing electromagnets across narrow air gaps. Since there is no motor attached to the system, everything relies purely on balancing those magnetic forces without disturbances from external forces. Within this system, the mechanical rotation comes down to Newton's second law factoring in beam inertia, pivot friction, and air drag, while the electrical portion of this system treats coil inductances as inversely proportional to the air gaps.





What is a PID Controller?

A PID controller is a widely used feedback control loop mechanism used in industrial and engineering applications. It continuously calculates an error value which is the difference between a desired setpoint and a measured process variable. It applies a correction based on proportional, integral and derivative terms.

- Proportional (P): This term produces an output value that is proportional to the current error value. If the error is large, the control action is large: if the error is small, the correction is subtle.
- Integral (I): This term accounts for past errors by accumulating them over time. It helps eliminate steady-state errors by continuing to apply correction until the error is completely zeroed out.
- Derivative (D): This term predicts future errors based on the current rate of change. By looking at how fast the error is changing, it acts as a damper to reduce overshoot and improve overall system stability.

How does it relate to the system?

In the context of the balancing beam , the open-loop plant is inherently unstable due to negative stiffness. With that, active feedback control is required to stabilize the beam at its equilibrium point. The block diagrams in the documents used illustrate a digital controller block utilizing an observer and state feedback that evaluates the error between the setpoint and the measured angle. It computes a control-current command to drive the state back to the origin, successfully stabilizing the system against its natural instability.

Creating The Desired System

Finding ζ and ω_n:

ζ	0.5	0.6	0.7	0.8	1.0
overshoot	16.3%	9.5%	4.6%	1.5%	none

$$t_s \approx \frac{4}{\zeta\omega_n}$$

ζ *is* damping constant and ω_n is the natural frequency. Using the damping ratio and natural frequency derived from overshoot percentages and settling time allow us to define the systems performance.

Construction of the desired equation:

$$s^2+2\zeta\omega_n s + \omega_n^2$$

The dominate complex conjugate pair of poles.

$$(s+p)$$

The non-dominate real pole located at –p.

$$D_{des}(s) = (s+p)(s^2+2\zeta\omega_n s + \omega_n^2)$$
$$D_{des}(s) = s^3+(p+ 2\zeta\omega_n)s^2 + (\omega_n^2 + 2p\zeta\omega_n s + p\omega_n^2)s$$

P=Poles

The Construction of The Real System

- Express the PID controller C(s) and the plant G(s) in terms of their respective numerators and denominators.

$$C(s) = \frac{K_d s^2+K_p s+K_i}{s} = \frac{N_c(s)}{D_c(s)}, \qquad G(s) = \frac{-b}{s^2+Zs-a} = \frac{N_g(s)}{D_g(s)}$$

- To form the actual denominator, derive the actual closed-loop denominator using:

$$D_{cl}(s) = N_c(s)N_g(s) + D_c(s)D_g(s)$$

- Substitute the specific plant and controller polynomials into the equation and expand them to group terms by powers of s, yielding:

$$D_{cl}(s) = (K_d s^2 + K_p s + K_i)(-b) + (s)(s^2 + Zs - a)$$
$$D_{cl}(s) = s^3 + (Z - bK_d)s^2 + (-a - bK_p)s - bK_i$$

Final Equations For PID Gains

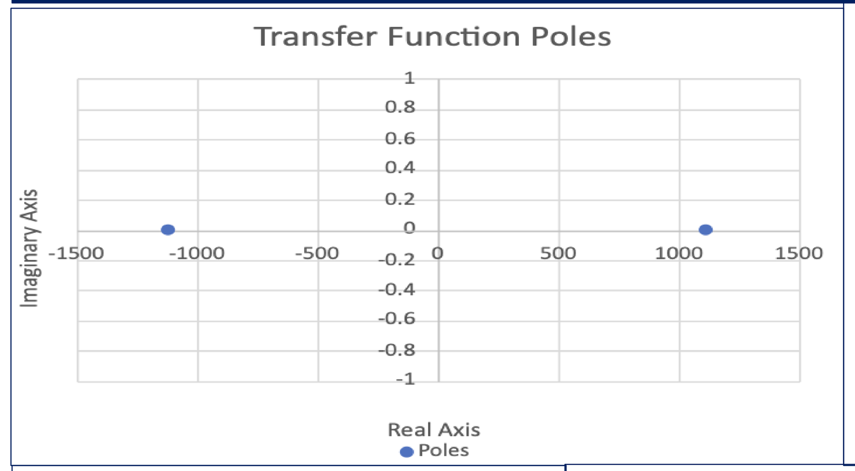
$$K_d = \frac{Z - p - 2\zeta\omega_n}{b}$$

$$K_p = -\left[\frac{p+\omega_n^2+2\zeta\omega_n p}{b}\right]$$

$$K_i = \frac{-p\omega_n^2}{b}$$

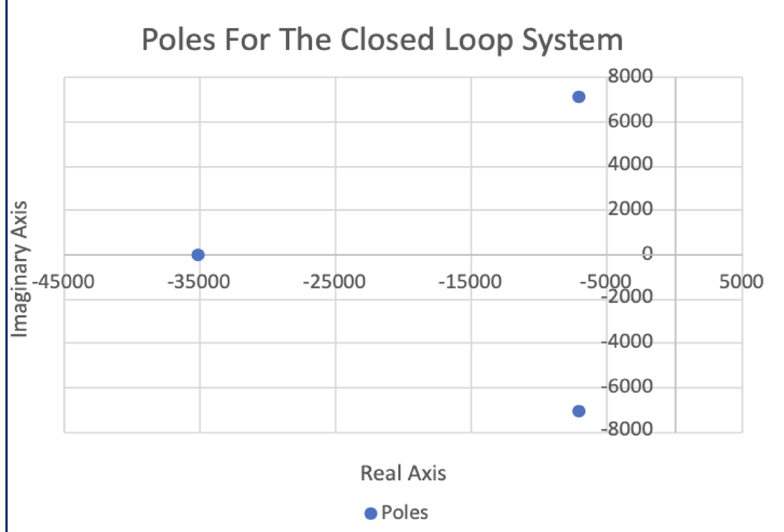
- Equate the coefficients of the actual closed-loop polynomial Dcl (s) to the coefficients of the desired target polynomial Ddes (s) term-by-term.
- Simultaneously solve the resulting equations to isolate and compute the final formulas for the PID controller gains

Plots For Poles



- This is the roots of our transfer function.

- These are the roots of our transfer function and controller combined.



Results

By calculating the Kp, Ki, and Kd gains, the system can meet specific performance requirements like settling time and minimal overshoot precisely. Because the PID controller adds an integrator to the second order balancing beam plant, the combined closed-loop system becomes third order. Meaning that the beam's motion is controlled by three poles: a dominant complex conjugate pair (which dictates the visible overshoot and oscillation of the beam) and a non-dominant real pole placed far to the left.

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